Conservation of Current is Universal and Exact

in five slides

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Maxwell's Magnetism

$$\mathbf{curl}(\mathbf{B}(x,t)/\mu_0) = \underbrace{\mathbf{J}(x,t) + \varepsilon_0 \frac{\partial \mathbf{E}(x,t)}{\partial t}}_{\textit{Current}}$$

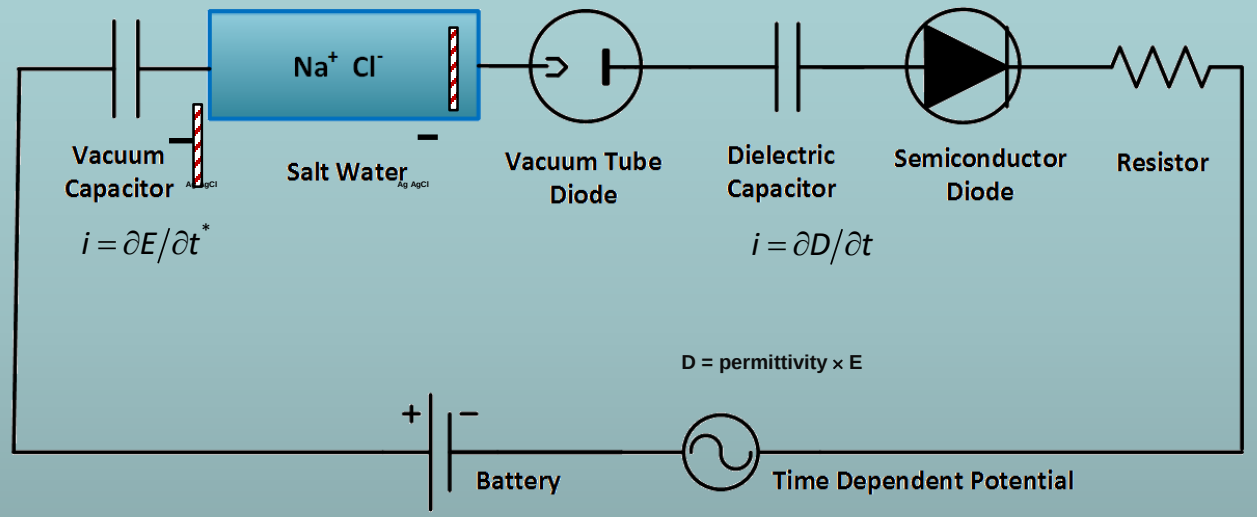
Current is Conserved

PERFECTLY

div curl $\equiv 0$ is an identity

$$\mathbf{div}\left(\underbrace{\mathbf{J}(x,t) + \varepsilon_0 \frac{\partial \mathbf{E}(x,t)}{\partial t}}_{\textit{Current}} \right) = \mathbf{0}$$

Current has very Different Physics
in Different Systems



but

Continuity of Current is Exact

$$J_{\text{Device 1}} = J_{\text{Device 2}} = J_{\text{Device 3}} \cdots$$

no matter what carries the current,
at all times and all locations!

Conservation of Current is Exact and Universal

$$\mathbf{div}\left(\underbrace{\mathbf{J}(x,t)+\varepsilon_0\frac{\partial \mathbf{E}(x,t)}{\partial t}}_{\textit{Current}}\right)=\mathbf{0}$$

even though
Physics of Charge Flow
Varies Profoundly

How can that possibly be?

$$\mathbf{div}\left(\underbrace{\mathbf{J}(x,t)+\varepsilon_0\frac{\partial \mathbf{E}(x,t)}{\partial t}}_{\textit{Current}}\right)=\mathbf{0}$$

Displacement Current
is
Different in Each Device

because

$\mathbf{E}(x,t)$ *is Different in every Device*

so the

TOTAL Current is exactly equal

at every time in every location and every device

Total Current = Displacement Current + Device Current

Electric Field takes on the Value that Conserves Current

$$\mathbf{E}(x,t) = -\frac{1}{\varepsilon_0}\int \mathbf{J}(x,t)dt$$

Specifically,

$\mathbf{E}$ changes the displacement current $\varepsilon_0\,\partial \boldsymbol{E}/\partial t$

So total current $\mathbf{J}(x,t)$ + $\varepsilon_0\,\partial \boldsymbol{E}/\partial t$ is always conserved

Details and PROOF
including quantum mechanics at
https://arxiv.org/abs/1609.09175