Dirk Gillespie

The Area Function

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The tangent line at $(x', R(x'))$ is

$$y = R'(x')(x - x') + R(x') \tag{1}$$

so that

$$x_0 - x' = -\frac{R(x')}{R'(x')}. \tag{2}$$

The height of the arc at x'' $(x \leq x'' \leq x')$ is given by

$$(x'' - x_0)^2 + h(x'')^2 = (x' - x_0)^2 + R(x')^2 \tag{3}$$

$$= \left(\frac{R(x')}{R'(x')}\right)^2 + R(x')^2 \tag{4}$$

or

$$h(x'') = \sqrt{\left(\frac{R(x')}{R'(x')}\right)^2 + R(x')^2 - \left(x'' + \frac{R(x')}{R'(x')}\right)^2} \tag{5}$$

$$= \sqrt{R(x')^2 - 2\frac{R(x')}{R'(x')} - (x'')^2}. \tag{6}$$

x is where $h = 0$:

$$x = \sqrt{R(x')^2 - 2\frac{R(x')}{R'(x')}}. \tag{7}$$

Then $A(x)$ is the area of the surface of revolution:

$$A(x) = 2\pi \int_x^{x'} h(x'') \sqrt{1 + \left(\frac{dh}{dx''}\right)^2} dx''. \tag{8}$$

The limit $R'(x') \to 0$ is well-defined and in that case $A(x)$ is the cross-sectional area of the cylinder

$$\pi R(x)^2. \tag{9}$$