A supplement to Bob's article: All spheres model of the L type calcium channel

Weishi Liu

August 23, 2014

This supplement explains briefly analytical results related to several parts of Bob's article. The same headings are used.

One dimensional models and baths.

Bob made it clear that one dimensional models with *constant radius* would be too simple to model channels and baths. This statement is also supported from a different angle – discussed below – by showing how *variable cross-section* could address important issues that *constant radius* fails to.

Variable cross section.

The one-dimensional models with *variable cross section* do have the ability to address issues concerning one-dimensional models that Bob raised in his article. This can be seen *easily* and *clearly* from, for example, the 2007 SIAM paper of Eisenberg and Liu cited in Bob's document where two ion species are considered. Specifically, the outer or slow system, which is valid for flows in the baths, is given by

$$\frac{d\phi}{dx} = \frac{\beta J_2 - \alpha J_1}{\alpha(\alpha+\beta)h(\tau)c_1}, \quad \frac{dc_1}{dx} = -\frac{\beta(J_1+J_2)}{(\alpha+\beta)h(\tau)}, \quad \frac{dJ_1}{dx} = \frac{dJ_2}{dx} = 0, \quad \frac{d\tau}{dx} = 1,$$

together with $\alpha c_1(x) - \beta c_2(x) = 0$, where x is the one-dimensional spatial variable, ϕ is the electric potential, c_1 and c_2 are the concentrations of the two ion species, J_1 and J_2 are the fluxes of the two ion species, $\alpha > 0$ and $-\beta < 0$ are the valences of the two ion species, $h(\tau)$ is the cross-section area over $x = \tau$. This is the system (19) in Eisenberg and Liu's paper.

The system clearly says that: the **RATES** of changes of electric potential ϕ and concentrations c_1 and c_2 are INVERSELY proportional to the **cross-section area** $h(\tau)$. That is, if one chooses *large* radius for bath portions, then the changes of electric potential and concentrations over bath portions would be *small*; in particular, if the radius for bath portions is "infinity", then the changes of electric potential and concentrations over bath portions could be ignored. **There is more**. Consider the following expressions for fluxes (it is system (23) in Eisenberg and Liu's paper)

$$
\begin{aligned}
J_1 =& \frac{(c_1^L - c_1^{a,l})}{\int_0^a h^{-1}(s)ds} \left(1 + \frac{\alpha(\phi^L - \phi^{a,l})}{\ln c_1^L - \ln c_1^{a,l}} \right), \\
J_2 =& \frac{(c_2^L - c_2^{a,l})}{\int_0^a h^{-1}(s)ds} \left(1 - \frac{\beta(\phi^L - \phi^{a,l})}{\ln c_2^L - \ln c_2^{a,l}} \right),
\end{aligned}
$$

where the bath to the left of the channel can be viewed as a subinterval of $[0, a]$ from $x = 0$, and ϕ^L, $\phi^{a,l}$, c_j^L and $c_j^{a,l}$ are some intermediate variables whose specifics are not relevant for the purpose of present discussion (see Eisenberg and Liu's paper for details). What is important is the denominator $\int_0^a h^{-1}(s)ds$. The integral implies that the bath portion, over which the cross-section area $h(s)$ is *large*, has *small* contributions to the integral and hence to fluxes. Again, if the radius for the bath portion is "infinity", then the contribution of the bath portion to the fluxes can be ignored; that is, the fluxes are mainly characters of ionic flows in the channel portion.

Four electrode methods.

The experimental design using *four electrode methods* is perfectly consistent with the analysis from one-dimensional models with *variable cross-sections*. More precisely, if the outer pair of electrodes were able to implement Dirichlet boundary conditions with *electroneutrality*, then there will be NO boundary layers. Based on the above discussion, if the radius over the bath potion (say, between the outer pair of electrodes and the inner pair of electrodes) is *large*, then the potential difference between the inner pair of electrodes (the 'voltage' for the current-voltage curve in experiments) would be *essentially the same* as that between the outer pair of electrodes (the 'voltage' for the current-voltage curve in models) .

One cannot expect, for experiments, a perfect electroneutrality boundary condition at the outer electrodes. In this case, the analysis shows that there will be a boundary layer correction. The end point of the layer will provide a REDUCED boundary condition that satisfies electroneutrality. If the radius over the bath potion is *large*, then the potential difference between the inner pair of electrodes would be *essentially the same* as that of REDUCED potential – so the boundary layer, if any, near the outer pair of electrodes becomes IRRELEVANT.

The design of four electrode methods is SO brilliant !!!