Electroneutrality

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Consider a sphere radius a.

Capacitance is $4\pi\varepsilon_0 a = (1.11\times 10^{-10})a$

Capacitance can be derived from Coulomb's law and Gauss' law with surface a sphere at infinity.

Charge for 1 volt is $4\pi\varepsilon_0 a = (1.11\times 10^{-10})a$

Number of unbalanced charges for 1v is $\dfrac{4\pi\varepsilon_0 a = (1.11\times 10^{-10})a}{1.61\times 10^{-19}} = 6.951\times 10^{8}\times a\,\text{charges}$

In a dielectric of dielectric constant ε_r, $6.951\times 10^{8}\times a\,\text{charges}\times\varepsilon_r$

Consider a 1 cm radius sphere of a 1 molar solution dielectric constant 78.8

Number of unbalanced charges is ~5.57×10^{8} charges for 1 v,

roughly 10,000,000 charges for 0.2 v in 1 cm sphere

<u>Now, compare to total number of charges in a typical system</u>

Volume of 1 cm radius sphere is $\tfrac{4}{3}\pi a^3 = 4.19\times 10^{-6}\ \text{m}^3$

Number of ions in 1 cm radius sphere of 1 molar solution is

$$4.19\times 10^{-6}\ \text{m}^3\times 6.02\times 10^{23}\,\frac{1}{\text{liter}}\,\frac{10^3\text{liter}}{\text{m}^3} = 2.52\times 10^{21}\ \text{ions.}$$

Unbalanced charges are a small fraction, namely $\dfrac{5.57\times 10^{8}}{2.52\times 10^{21}} = 2.2\times 10^{-13}$ for 0.2 v in 1 cm sphere.

General formula (for 1 molar salt at 1 v) is $\dfrac{4\pi\varepsilon_r\varepsilon_0 a}{\tfrac{4}{3}\pi a^3 N_A} = \dfrac{3\varepsilon_r\varepsilon_0}{a^2}\dfrac{1}{N_A} = \dfrac{3\times 78.8\times 8.85\times 10^{-12}}{a^2(6.02\times 10^{23})} = \dfrac{3.47\times 10^{-33}}{a^2}$