Mass Action
and
Conservation of Current

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Abstract

Electronic technology has remade our world in fifty years, increasing capability by factors of billions, unprecedented in human history. Electronics depends on models that precisely represent its devices and circuits. Circuit models are robust and work well over a wide range of conditions without changing parameters.

Circuit models are based on the laws of electricity, conservation of charge, and Kirchoff's current law: current flow is exactly the same everywhere in a series of devices (with two terminals, like diodes or ionic channels). Interruptions anywhere stop current everywhere, even far away. Succinctly, for steady flows $\nabla \cdot \vec{\mathbf{I}} = 0$, where $\vec{\mathbf{I}}$ is the vector field of current $\vec{\mathbf{I}} = \sum_k z_k F \vec{\mathbf{J}}_k$, z_k is the charge (valence) on one ion, F is the Faraday, and $\vec{\mathbf{J}}_k$ is the vector field of mass flux.

Chemical and biochemical models have been built on conservation of matter, expressed as the law of mass action, with constant rate constants. Chemical and biochemical technology has made striking progress by making compounds, not so much by making devices.

Device design in the chemical world is difficult because the rate constants of the law of mass action change from one set of conditions to another. The law is not robust in most cases and cannot serve the same role that circuit models do in our electrical technology.

The law of mass action does not automatically conserve current, as is clear from the mathematics of a simple case, chosen to illustrate the issues involved: Consider $X \rightleftharpoons Y \rightleftharpoons Z$ with difference $I_{XY} - I_{YZ} = z_X k_{xy} F[X] - z_Y k_{yx} F[Y] - z_Y k_{yz} F[Y] + z_Z k_{zy} F[Z]$ and $I_{XY} - I_{YZ} \neq 0$. Of course, there are some series of reactions in which current is conserved and thus the same everywhere. The Appendix identifies special symmetrical cases in which charge flow is conserved. The law of mass action, however, does not seem to force a series of chemical reactions to have the same current flow everywhere. Interruption of far-away current does not stop current everywhere in a series of chemical reactions (analyzed with law of mass action), and so does not obey Kirchoff's current law. The Appendix also evaluates consequences when current flow is not conserved

I fear classical models of many reactions will be in peril, when current flow is examined.

Variational methods have only recently been developed to ensure that charge flow is conserved globally, along with mass, in dissipative systems like ions in solution or proteins. The Energy Variational Approach *EnVarA* developed by Chun Liu, more than anyone else, should allow the development of robust models of chemical, biochemical, and biological systems.

I believe robust models and device designs in the chemical world will not be possible until the law of mass action and Kirchoff's current law are embedded together in a consistent variational model of energy and dissipation.

Scientific laws are sometimes vague, sometimes precise. Scientific laws are often residues of revolutions that once gave us new knowledge of the world around and within us. It is easy to continue to use scientific laws uncritically even after they have been overtaken by new knowledge. Scientific laws are usually learned early in our careers before we have refereed grants and papers, before our critical skills are honed and so have a life of their own, a momentum that is hard to change.

Conservation of Mass and Conservation of Charge and Current. I write to discuss two scientific laws: conservation of mass, and conservation of charge flow.

Both conservation principles seem precise laws of science and in historical context they certainly once were precise laws, not vague residues of previous revolutions. Both conservation of mass and conservation of charge/current are so widely used that each might be called the foundation of a field, chemistry and physics, respectively. Ionic solutions must satisfy them both.

Chemistry uses mass conservation almost everywhere, in the form of the law of mass action (derived using conservation of mass in eq. $(1)-(4)$ below). The law of mass action helps design, build, and understand new molecules, the main goal of classical chemistry. It is less useful when the goal is to interact and pass signals to the macroscopic world.

Physics uses conservation of charge everywhere it deals with electricity. Conservation of charge implies Kirchoff's current law, conservation of steady charge flow, $\nabla \cdot \bar{\mathbf{I}} = 0$. Time dependent current is conserved when current is generalized to include displacement current, as explained in textbooks [43, 83, 156]. Displacement current is "Maxwell's extension of Ampere's law" (p. 521 of [84]) needed to describe the source of the magnetic field. Maxwell's extension allows light to propagate in a vacuum. Kirchoff's current law says the flow of charge is continuous, without loss in circuits, once displacement current is included. Current is exactly the same everywhere in a series of two terminal devices. Interruption of current anywhere in a series circuit interrupts signal and current flow everywhere, even signals carried by currents far away from the interruption.

The main interest of electrical engineering is signals. Electrical engineering uses Kirchoff's laws to make models of its devices. Electronic devices behave in one way, with one model and one set of parameters, over a range of conditions because Kirchoff's laws are robust, indeed exact to some eighteen significant figures [156]. Without Kirchoff's laws, we would not have integrated circuits or digital devices to manipulate signals. We would not have computers, smartphones, and the video devices that have remade our world. These devices operate at very high speeds with electrical potentials changing volts in 10^{-9} sec. Devices operating at these speeds could not be described with one set of parameters without Maxwell's extension of the idea of current. Every time we use our smartphones or televisions, we depend on the properties of displacement current and the continuity of current flow, Kirchoff's law.

Biological systems involve both chemical reactions and charge in their devices. Biological systems are always embedded in ionic solutions, and nearly always involve chemical reactants and enzymes with electrical charge. Substrates of reactions catalyzed by enzymes are usually charged and are always embedded in solutions containing the ions Na^+, K^+, and Cl^- and involving Ca^{2+}, often as a variable that turns biological function on or off. Reactants almost always flow in biological systems. Equilibrium and death are nearly synonymous in life. Biological systems must satisfy conservation of matter and conservation of current.

We face a problem when we try to apply both conservation laws together. The laws do not always agree. Indeed, they often conflict.

My goal is to make models involving ions in chemistry, biochemistry, and biology as successful as the models of electronic devices. We need robust theories (or simulations) of signals in chemical reactions in ionic solutions that successfully use one set of parameters in a range of conditions, and include the global properties of the electric field. Models of signals in electrochemical devices and technology as robust as those of electronics would make a great difference. I believe that goal can only be reached after the law of mass action is extended to embed Kirchoff's current law. Only then will current interrupted anywhere interrupt current everywhere (in a series of reactions).

The Law of Mass Action has been a keystone of classical chemistry. It was developed to deal with isolated systems, originally with perfect gases [4, 58, 117, 131, 135], and allowed chemists to deal with molecular and atomic reactions before physicists were convinced that atoms existed. The law of mass action helps chemists build molecules of great use and importance.

But isolated chemical reactions must contact the outside world to pass signals and interact with it, just as electronic systems contact the outside world and pass signals through inputs and outputs. Biological systems contact surrounding solutions and cells through ion channels and transporters. The law of mass action was not designed to contact the outside world. It was designed to help chemists build and understand individual molecules. Signals and connections with the outside world almost always involve electricity because charge flows through the contacts that connect chemical reactions with the outside world. The contacts are usually the boundary conditions of mathematical models.

Engineering deals with systems that are not isolated. Its devices contact the world through power supplies and inputs and outputs. Devices have outputs that follow inputs according to simple rules, for example, the output of an amplifier follows the input according to the gain. The input output rules are derived from Kirchoff's current law.

Biology deals with systems that are not isolated. They usually have inputs and outputs and are driven by concentration gradients that are power supplies. Biological systems interact with surrounding solutions, cells, and tissues.

I argue that the law of mass action must be extended to deal with inputs and outputs and electrical charge if theories and simulations of nanodevices (technological or biological) are to be useful in more than one set of conditions.

'The Law of Mass Action does not conserve current' seems an unlikely statement and so mathematical proof is desirable before verbal discussion. We examine a simple case using high school algebra and a little calculus and ask does it conserve current? Is current the same everywhere in the series of reactions?

The same questions can then be asked of whatever series of reactions are of interest. Sometimes the answer will be current and mass are both conserved. More often, the answer will be, they are not, I suspect.

Eq. (1) shows the reactions we analyze.

$$X \underset{k_{yx}}{\overset{k_{xy}}{\rightleftarrows}} Y \underset{k_{zy}}{\overset{k_{yz}}{\rightleftarrows}} Z \tag{1}$$
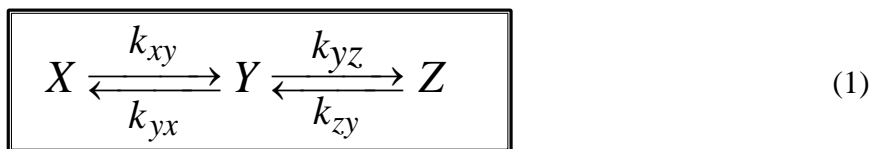
We define 'law of mass action' for this paper as just eq. (1) & (4) with symbols defined below. The rate constants in eq. (1) & (4) are taken as constants and thus of course are independent of each other. If the law were robust, the rate constants found experimentally under one set of conditions would be found under another set of conditions. Such is sometimes the case, but not very often [33, 34].

Generalizations of rate constants are sometimes made but, as we discuss below—in the section 'How to extend the law of mass action?'—the generalizations common in the literature do not allow the law of mass action defined here by eq. (1) & (4) to satisfy Kirchoff's current law globally. The global nature of the electric field (illustrated in Fig. 2 and its discussion) allows remote devices and boundary conditions to change local atomic flows. Indeed, interruption of current in a series of reactions stops current everywhere, even far away. I fear many series of reactions found in the literature will not have the same current everywhere, when the current flow is determined explicitly.

Of course, there are special cases in which the law of mass action can conserve current flow exactly and those in fact may be the ones where it has proven most quantitatively useful. A number of reactions may conserve current approximately. In other cases, like eq.(1), the reaction will not conserve current, even approximately. Each case needs to be studied separately.

The central fact—that applies to any chemical reaction, not just eq. (1) & (4)—is that the global realities of the electric field need to be embedded in the atomic scale treatment of the reaction. These global realities have not been dealt with in the literature, as far as I can find. I do not know cases where rate constants have been generalized so interruption of far-away current interrupts current everywhere in a series of reactions.

PROOF. The current flow in the reactions of eq. (1) is easily shown to be

$$I_{XY} = Fz_X \cdot k_{xy} [\![X]\!] - Fz_Y \cdot k_{yx} [\![Y]\!]$$

$$I_{YZ} = Fz_Y \cdot k_{yz} [\![Y]\!] - Fz_Z \cdot k_{zy} [\![Z]\!] \tag{2}$$

Units for net current[†] I_{XY} are (cou/liter)/sec = cou/(liter sec). Unidirectional flux J_{xy} are (moles/liter)/(sec) = moles/(liter sec). Double brackets like $[\![X]\!]$ indicate activities, the generalization of concentration (number density) needed in biological solutions, as discussed below. Units for rate constants units are $\left(moles/(liter\,sec) \right) / \left(moles/liter \right) = 1/sec$. The valences (i.e., charges) of each reactant are z_X or z_Y. F is Faraday's constant.

[†] Unidirectional fluxes use lower case subscripts. Net fluxes use upper case subscripts and are marked with an over-bar. Upper case subscripts are used for currents to emphasize the distinction between net currents and unidirectional fluxes. Note that unidirectional currents are not defined or used in this paper.

It is clear that $I_{XY} \neq I_{YZ}$.

In general, Kirchoff's current law is violated:

$$\boxed{I_{XY} \neq I_{YZ}, \text{ for a range of concentrations, rate constants, or charges.}} \qquad (3)$$

<u>Details of Proof</u>: Rate equations equivalent to chemical reactions involve net flux like $\bar{J}_{XY}$. The chemical reactions in eq. (1) defines the net fluxes, units moles/(liter sec),

$$\begin{aligned} \bar{J}_{XY} &= k_{xy} [\![X]\!] - k_{yx} [\![Y]\!] \\ \bar{J}_{YZ} &= k_{yz} [\![Y]\!] - k_{zy} [\![Z]\!] \end{aligned} \qquad (4)$$

with the definitions of rate constant k_{xy}, for example

$$k_{xy} = \frac{J_{xy}}{[\![X]\!]} = \frac{J_{xy}}{-\dfrac{d}{dt} [\![X]\!]_{J_{yx}=0}} \qquad (5)$$

<u>Remark.</u> Unidirectional fluxes J_{xy} are conventionally measured by tracers—originally radioactive isotopes, now usually fluorescent probes—flowing into an acceptor solution in which the concentration of tracer is zero. The concentration in the acceptor solution of the substance being traced is not zero in most cases. Note that the activity $[\![X]\!]$ depends on the concentration of all the other ions in a significant way in nonideal solutions [6, 9, 19, 28, 30, 31, 34, 42, 47-49, 51, 52, 79, 87-90, 92, 94, 95, 106, 109, 113, 127, 133, 150, 151, 157]. In nonideal solutions, changes in the concentration of one substance $[Y]$ change the activity of another substance $[\![X]\!]$ and also change its flow.

<u>Almost all biological solutions are nonideal.</u> Ionic mixtures or solutions of biology usually involve Ca^{2+} and are significantly nonideal, often with activity coefficients of the order 0.5, implying that half of the thermodynamic driving force of that ion is nonideal and depends on the concentrations of all ions, not just that ion, itself. In the language of biophysics, all ions interact with each other in nonideal solutions, like Ringer solutions, whether or not proteins, channels, or nucleic acids are present.
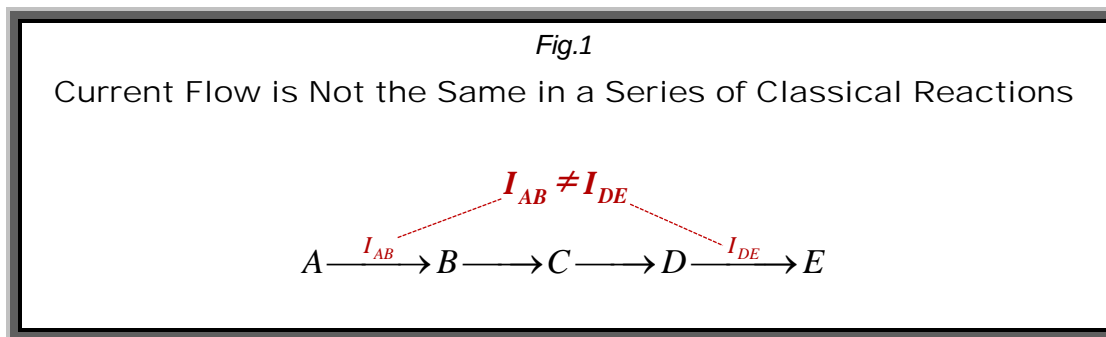
<u>Back to the proof.</u> The proof of eq. (2) is completed by writing net fluxes like $\bar{J}_{XY} = J_{xy} - J_{yx}$ as the difference of unidirectional fluxes. Net flux is converted into net current like I_{XY} using the proportionality constant zF (with z for the species in question) between flux and current.

$$I_{XY} = Fz_X \cdot k_{xy} [\![X]\!] - Fz_Y \cdot k_{yx} [\![Y]\!]$$

$$I_{YZ} = Fz_Y \cdot k_{yz} [\![Y]\!] - Fz_Z \cdot k_{zy} [\![Z]\!] \qquad (6)$$

Obviously, $I_{XY} \neq I_{YZ}$.

This ends the proof.

The main result of eq. (2) & (3) is simple: the law of mass action conserves current only under special symmetric circumstances. Examples are shown in Appendix eq. (9). I fear problems may be found in many of the reaction schemes in the literature—with substantial effect Appendix (10) – (12)— when those reactions are examined as we do here, to see if the same current flows in every reaction in the series.

Fig.1

Current Flow is Not the Same in a Series of Classical Reactions

$$I_{AB} \neq I_{DE}$$

$$A \xrightarrow{I_{AB}} B \longrightarrow C \longrightarrow D \xrightarrow{I_{DE}} E$$

<u>Organization of paper.</u> This paper begins by trying to motivate the treatment of chemical reactions as devices. The properties of charge flow have not been widely discussed in classical reactions (as described by the law of mass action), so I use mathematics to show that current is not conserved. I treat a simple but widely used case, hoping that this will be more convincing than words and easier to extend to other reaction schemes. The methods used can be applied to any series of reactions to see if they satisfy both conservation of current and conservation of mass.

A general discussion follows that is designed to explain verbally why current cannot be conserved in many applications of the law of mass action. The abstract nature of current is illustrated with many examples, discussed in some detail, since this crucial idea is not widely known to scientists, in my experience. Finally, the Appendix eq. (10)-(12) gives symmetrical conditions in which the law of mass action can conserve current. Asymmetric conditions are shown to produce charge imbalance likely to have noticeable effects.

<u>Logic of current flow.</u> Conservation of charge and charge flow implies that the current is identical in each reaction in the series of reactions of Fig. 1, eq. (1), or of Fig. 2: the flow of charge is continuous without loss. Interruptions anywhere stop current everywhere in any series system. If current is forced to be zero in one reaction, it must be zero in every reaction, because Kirchoff's current law says current is the same everywhere in a series of reactions, just as it is in a series of resistors.

Models of series reactions should also have continuous current, the same everywhere.

<u>'Completing the circuit'</u> is a necessity familiar to all who have wired up a circuit—the lights on a Christmas tree—even if they do not know the principle 'Kirchoff's current law'. The flow of charge is continuous on all scales because conservation of charge and its movement is true on all scales. Interrupting current on an atomic scale (in a semiconductor diode, or an ionic channel) interrupts current on the macroscopic scale in the inputs and outputs of the diode, or in the solutions and cells connected to the ionic channel.

<u>Logic of classical reactions.</u> Chemical reaction schemes usually have rate constants that are independent of each other, so interruption of flux in one reaction in the scheme does *not* change rate constants elsewhere in a model of a series of chemical reactions. Reactants can accumulate and build up mass in that model of reactions. The accumulated mass changes flow in a complex

way, different in different systems, that have different constitutive equations. The accumulation of mass does not follow a universal rule. Classical schemes follow conservation of mass, not conservation of current.

Accumulated charge is much simpler than accumulated mass. Accumulated charge has universal properties, independent of the physical nature of the charge. Particle and quasi-particle currents that accumulate at a junction (with sum i_Σ) change the time derivative of electric potential by an invariant equation $\partial V/\partial t = i_\Sigma/C$ (determined by the Maxwell equations). The change in electrical potential carries away a displacement current

$$i_{displacement} = C(\partial V/\partial t) = i_\Sigma \tag{7}$$

This displacement current is **exactly** equal to the sum i_Σ of the currents flowing into the junction, and is equal to $C(\partial V/\partial t)$ without known error, to about one part in 10^{18}. Kirchoff's current law is exact if current is re-defined to include $i_{displacement} = C(\partial V/\partial t)$ no matter what the constitutive laws. The displacement current (and equivalently $\partial V/\partial t$) 'take up the slack' so that Kirchoff's current law (using the extended definition of current) is exact. No charge accumulates at all beyond that defined by the integral of the displacement current.

Current is redefined to include displacement current (see textbooks [43, 83, 139, 156]) because the redefined current is the source of the magnetic field, as Maxwell discovered.

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \overbrace{\varepsilon_0 \frac{\partial \vec{E}}{\partial t}}^{\substack{\textit{Displacement} \\ \textit{Current}}} \right) \tag{8}$$

$\vec{B}$ is the magnetic vector field; μ_0 is the magnetic constant, the magnetic 'permeability' of a vacuum; ε_0 is the corresponding 'electrostatic constant', the permittivity of free space. It is mysterious that magnetic charges do not exist, i.e., $\nabla \cdot \vec{B} = 0$. Magnetism only arises from current flow $\vec{J} + \varepsilon_0 \partial \vec{E}/\partial t$.

Accumulation of charge is special because it is universal. The precise linkage between potential change, charge accumulation and displacement current is a special feature of electromagnetism because it is universal. It is a property of a vacuum, the constitutive equation of a vacuum, if poetic license is allowed. The displacement current allows electricity and magnetism to combine into the propagating radiation we call sunlight as it flows through the vacuum between sun and earth with velocity $c = 1/\sqrt{\mu_0\varepsilon_0}$. No universal precise linkage exists between mass accumulation, rate of change of chemical potential, and flux, for example. That linkage depends on the details of the constitutive equation, and on all components of an electrolyte, if the system is a nonideal ionic solution, like those that life requires.

Extensions of mass action. Charge movement at power supplies, inputs, and outputs are involved as devices contact the outside world. Charge flow must be included if a chemical reaction connects to the outside world through electrical inputs and outputs like those used in our engineering technology and that current should follow Kirchoff's current law.

If the law of mass action of a particular reaction does not follow Kirchoff's law I do not see how it can be expected to be a robust model of that reaction. In that case, it seems that the law of mass action should be extended.

Historically, current flow has not been included explicitly in series chemical reactions in my opinion because the mathematics needed was not available until recently. Mathematics that forces descriptions of chemical reactions to conserve current and charge in dissipative systems (like ionic solutions) has been developed only in the last few years as energy variational principles have been developed by Chun Liu, more than anyone else.

Energy Variational Approach. The energy variational approach *EnVarA* is defined by the Euler-Lagrange process [41] as generalized by Chun Liu, and colleagues, into an energy/dissipation functional. The generalized functional combines two variations yielding a single set of Euler Lagrange equations (in Eulerian coordinates of the laboratory) using push back and pull forward changes of variables [35, 46, 136, 152, 154]. In this way, *EnVarA* can deal with dissipation (friction) and ionic solutions.

EnVarA describes conserved energy using the classical Hamiltonian variational principle of least action described in textbooks of mechanics. It deals with friction using the Rayleigh dissipation principle described in textbooks of irreversible thermodynamics. When combined, these principles allow energy to be degraded into entropy as matter and charge flow in frictional materials [155], like electrolyte solutions.

Ionic solutions experience friction because an ionic solution is a condensed phase essentially without empty space. Ice floats on water, so liquid H_2O has greater density than solid H_2O and presumably less space between its atoms than the solid. Atoms and molecules in a condensed phade cannot move without colliding. Collisions randomize originally correlated motions and make them into the kind of randomized motion that we call heat [10]. The variance of the displacement is what we call temperature. The macroscopic names for the conversion of translational to randomized zero mean (nearly) Brownian motion, are dissipation and friction.

Skepticism and History: Most readers will be understandably skeptical of a conflict between long established laws: how has the law of mass action produced such useful chemistry, over so many years, if it does not conserve current and charge?

The answer is that chemistry has been interested in molecules. Chemists make molecules, not signals. Engineers have been interested in signals, and charges, not molecules.

In chemistry the law of mass action has been used build molecules, not to pass signals to or interaction with the outside world. In classical chemistry, the law of mass action has been used as a productive guide to understand and build molecules, but not as model of inputs and outputs for designing signals and devices.

In classical biochemistry, the law of mass action has been used in another clever and ingenious—and biologically wise—way to build useful models, despite their limitations. A senior biochemist recently told me "The art of biochemistry is to study enzymes in conditions that give insight to biological function, even if the conditions are not just those inside cells. The art is to choose experimental conditions in which biochemical reactions are well behaved and rate constants resemble those in real biological systems, so results are practically useful."

The biochemists' use of mass action has been expedient, practical, admirable, and a productive response to human limitations, I think. A scientist should do what she/he can do, even if it is much less than what she/he wishes to do.

We now discuss charge movement so readers less familiar with electric circuits will know what I am talking about when I discuss current through a vacuum capacitor, for example.
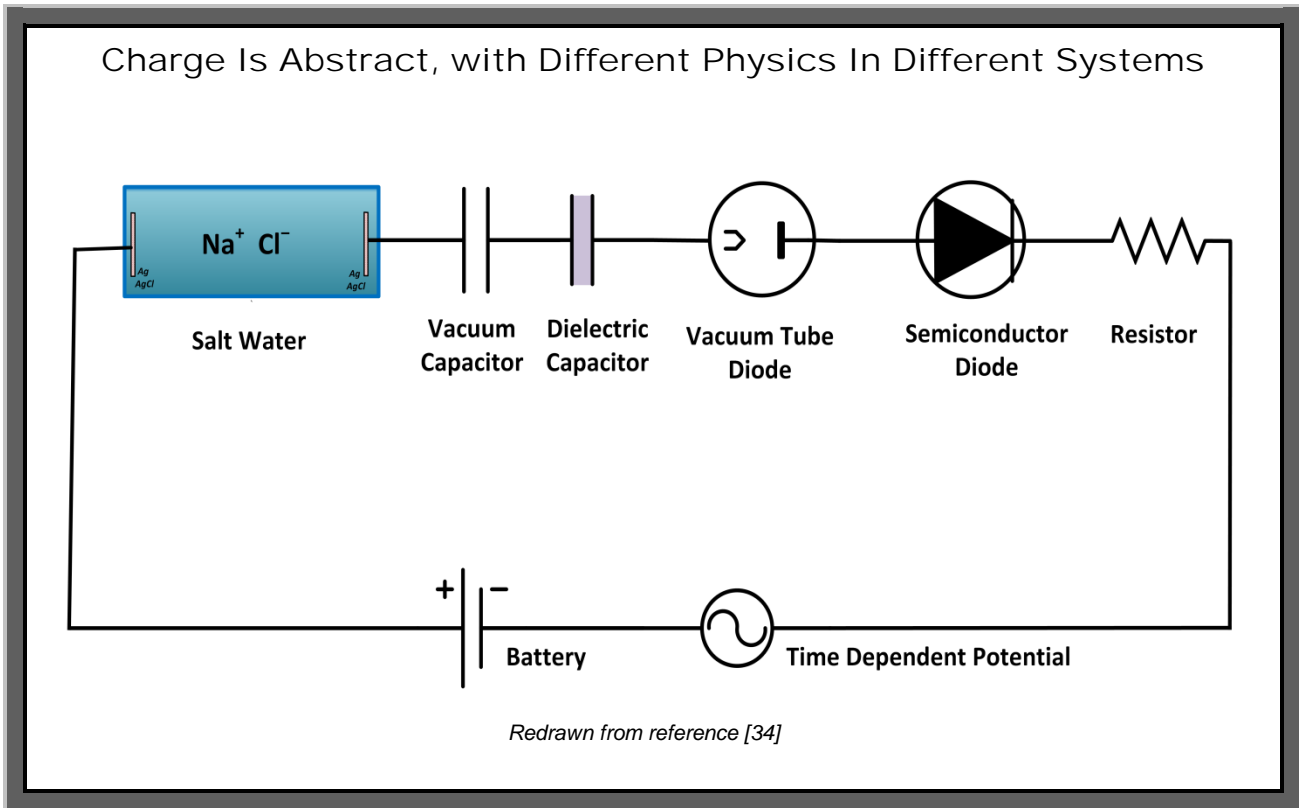

Fig.2. We see that charge in one physical system is quite different from charge in another. Charge flow is **not** simply the physical movement of particles of definite charge (and mass). Charge flow in a vacuum capacitor, in an ionic solution, in a wire are all quite different. Current is not just the movement of ions or electrons or protons. Note that the electric charge in different devices varies on very different scales from subatomic (in dielectrics) to atomic (in diodes, ionic conductors, etc.) to macroscopic. Thus, treatments of charge must be multiscale.

<u>Charge is an abstraction that is conserved.</u> Physicists teach that conservation of charge and current are universally true, exact from very small to very large scales and very small times to very large times.[†] What physicists often do not teach clearly is that charge is an abstract idea.

Many students believe that charge is always carried by particles and so conservation of mass (of charged particles) implies conservation of charge and current. That is not true. Charge is carried between the plates of a vacuum capacitor as a displacement current that does not involve the movement of particles. [43, 83, 139, 156] Displacement current is $\vec{\mathbf{J}}_D = \varepsilon_0\, \partial\vec{\mathbf{E}}/\partial t$, see eq. (8). Current flow in semiconductors is carried by mathematical fictions called quasi-particles that move according to classical physics [78, 111, 115, 142, 149], not involving solutions of Schrödinger's equation. This is an important property of semiconductors [84, 86], crucial to their use.

[†] Even the Casimir effect of quantum physics [130] is seen as a property of Maxwell's equations by some [80].

An essential idea of electricity and magnetism as explained in textbooks [83, 139, 156] is that charge flow is continuous (without loss in a series circuit) no matter what the physical nature of the charge. Saslow's [139] treatment of continuity of current in circuits is to my taste, careful, helpful, and straightforward.

Fig. 2 tries to show this idea in a concrete way that anyone can build in their lab. 'Completing the circuit' implies that current in every device is the same. Continuity of current (i.e., Kirchoff's current law) says that time-varying currents are the same in any series of devices even if they have very different physics and different constitutive laws, even if they involve chemical reactions (Fig. 1 and eq.(1)).

Ionic Conductor. The ionic conductor in Fig. 2 is a cylinder containing Na^+Cl^-. Here, current flow (at a frequency say of 1 Hz) is almost entirely the physical movement of charged particles, of ions, say sodium and chloride ions and follows simple constitutive laws (when concentrations are $< 2 \times 10^{-2} M$ and flows are not large enough to change concentrations). These ions are hard spheres. The finite size of these spheres is significant in the ionic mixtures found everywhere in biology and in general at concentrations greater than say $2 \times 10^{-2} M$. The finite size makes constitutive equations (valid at all concentrations of mixtures of different types of ions) much more difficult than the classical constitutive equations for quasi-particles that are points. Finite size implies saturation effects—space cannot be filled more than once—and these imply that 'everything interacts with everything else'.

Numerical difficulties in dealing with spheres are substantial. Spheres must be computed in three dimensions because spheres do not exist in one and two dimensions. That is to say, objects with a single radius have very different surface to volume ratios in one, two and three dimensions and so fill space very differently. Phenomena in which spheres fill a significant fraction of three dimensional space are not easily approximated in one or two dimensions. Computation of the forces that prevent overlap of spheres is difficult because those forces are strong and vary steeply in three dimensions. Bottom line: nonideal solutions remain a challenge, as documented below.

Vacuum Capacitor. We move to a vacuum capacitor, in which the space between the two plates is completely empty of matter (as it would be in outer space, for example). The current flow through this capacitor is just as real as the movement of ions of Na^+Cl^- in the cylinder even though no particles or spheres are present, and no mass moves at all.

The displacement current between the plates of the capacitor is a property of the electric field itself, as explained in textbooks of electricity and magnetism [43, 83, 139, 156] and is described by the exact and simple constitutive equation $i_{displacement} = C(\partial V / \partial t)$, where the displacement current $i_{displacement}$ (amps) is strictly proportional to the capacitance C (farads) and the time rate of change $\partial V / \partial t$ of the voltage across the capacitor. Unlike other constitutive equations, the constitutive equation for vacuum current is exact, valid to some eighteen significant figures.[156]

This displacement current induces a magnetic field just as current carried by ions produces a magnetic field. Indeed, without displacement current in a vacuum, Maxwell's equations do not allow sunlight to propagate through the vacuum of space. With this exact expression for displacement current, light propagation is a solution of the Maxwell equations, and in fact the speed c of propagation of light can be computed from measurements of electrical and magnetic constants, entirely independent of measurements on light itself, by Maxwell's remarkable formula $c = 1/\sqrt{\mu_0 \varepsilon_0}$. Light propagates according to Maxwell's equations over

astronomical distances, so we know that the constitutive equation that calculates that speed must be accurate to many significant figures.

Dielectric Capacitor. In the dielectric capacitor in Fig. 2 (filled with real material, for example the plastic = Polytetrafluoroethylene = PTFE = Teflon), current flow is more complex, and involves the effect of an applied electric field on the spatial distribution of the electric charge intrinsic to the atoms, molecules, and substance of the dielectric. ('Intrinsic' here means the distribution of charge present when there is no applied electric field.) Note that the electric charge in a dielectric, or an ionic solution, or a protein or nucleic acid, for that matter, varies significantly on all scales from subatomic to macroscopic.

The properties of dielectrics cannot be described in detail here because they vary so much with material, voltage, and time. But some properties need emphasis. Intrinsic charges do **not** move from plate to plate in a real capacitor. The current that flows from plate to plate and within the dielectric is a dielectric displacement current not carried by the movement of mass (any substantial distance). The current in a real capacitor is an abstraction. It is the sum of the vacuum displacement current and the material displacement current (in the dielectric) produced by the distortion of intrinsic charges. The material displacement current describes the movement of a charge on a nonlinear time dependent spring (with damping) that returns to its original position after being perturbed by an external electric field for a finite time. Calculating this material displacement current involves solving a nonequilibrium time dependent version of the Schrödinger equation, a difficult task, involving a macroscopic number of atoms and millisecond time scales in cases of biological interest.

Ideal dielectrics do not approximate ionic solutions. Scientists have avoided the difficulties of solving the Schrödinger equation in a macroscopic system like a dielectric, or ionic solution, by using approximations. They discuss an ideal dielectric with properties independent of field strength and independent of time. The approximation over a wide range of electric field strength is satisfactory in most materials.

The approximation of a time independent ideal dielectric is poor over a wide range of times. For example, most solutions of ions in water need effective dielectric coefficients to describe the polarization charge induced by permanent charges. This dielectric charge changes—after a step electric field is applied—from about 2 to about 80 as time moves on, from zero, to 10^{-15} sec to say 10^{-5} sec and the change in dielectric charge depends on the substance.[5] There is not a universal constitutive law or approximation for the time dependence of dielectric properties.

A factor of 40 change with time is not small. Multiscale analysis must deal with this multiscale problem because atomic motion has macroscopic consequences on all time scales.

The time dependence of real dielectrics needs to be dealt with in biological applications: ideal dielectrics do not approximate the properties of ionic solutions in which biology occurs. The time dependence of polarization charge in proteins has been extensively studied [123-125] and varies over the whole time scale from atomic to macroscopic. It often has slow components, 10^{-3} sec or slower.

Biology is controlled by atomic structures that move significantly in 10^{-15} sec but have dramatic effects on biological functions 10^{-5} sec to 10 sec later. The time dependence of these induced polarization charges appear in simulations (with atomic resolution) as changes in the

orientation and induced polarization of water molecules, and the distribution of ions, as well as the distribution in proteins of permanent charges, permanent dipoles, and induced dipoles.

Atomic scale simulations must last a long time. Atomic scale simulations seeking agreement with experimental data—available for a wide range of solutions [5], and proteins [123-125]—must last long enough to account for the discrete charge movements that produce an effective dielectric coefficient of 80, as measured experimentally. Otherwise, substantial changes in the electric field will not be seen in the simulation, even if those changes in the electric field have great biological significance.

Electric fields in biology are strong. The nerve signal is a change in the electric field of the greatest biological significance. Information transfer in the nervous system, coordination of contraction in muscle, including the coordination of cardiac contraction that allows the hart to pump blood, are all directly controlled by the action potential, the biological name for the nerve signal.

Electric fields during a biological action potential are slow (say 10^{-3} sec), and macroscopic in scale, spreading 10^{-3} meters, propagating 1 meter (in humans), and they are remarkably multiscale. Handfuls of atoms in a channel protein control the macroscopic propagation and the nerve signal itself is current carried by ions (that are single atoms) moving through those proteins.

The electric field of the action potential is strong, typically some 0.1 volts across a 2×10^{-9} meter thick membrane or a 3×10^{-10} meter selectivity filter (EEEE group in a calcium channel), 5×10^7 to 3×10^8 volts/meter. These electric fields distort the intrinsic distribution of charge within the dielectric on many scales of time and distance, including electrons inside molecules and atoms, They reorient polar molecules that have an intrinsic asymmetric distribution of charge. Charges only move a small amount in dielectrics in response to the applied electric field—reminiscent of the sloshing of tides in the ocean and on beaches on the earth created by the moon's gravitational field—and they eventually return to their resting position when the field is turned off but those small movements of charge produce large effects because the electric field is so strong.

Dielectric currents share some of the properties of vacuum displacement currents but they do not have a universal exact constitutive law, not even approximately. It is important not to confuse dielectric and vacuum displacement currents. It is important to realize dielectric displacement currents can be large and produce large effects in biology, as they do in semiconductors.

Nonlinear components of dielectric current are important in biology. Nonlinear components of dielectric displacement current have large effects important in biology (e.g., nerve [1] and muscle [141]) where they control enzyme function [91] as well as the opening and closing of the channels that produce the action potential [148]. These nonlinear dielectric (i.e., displacement) currents can be recorded as 'gating' currents because of the Shockley-Ramo theorem [122], which is Kirchoff's current law in another form. The dielectric currents involve a tiny fraction of all the charges in and near an ionic channel, far less than one per cent (see Appendix, "Size of Effects", near eq.(15)), but Kirchoff's current law and the strength of the electric field make gating currents measurable, easily in the hands of skilled experimentalists.

Any electric field is extraordinarily strong compared to diffusion. Tiny displacement currents can be recorded in routinely in many laboratories because the electric field is so strong. If it wee not so

strong, these tiny concerted movements of atoms would be lost amongst the Brownian thermal motion. As Feynman memorably mentions at the very beginning of his textbook [43], one per cent excess of charge in a person at arm's length produces force enough to lift the earth!

Concentration effects are very small. On the other hand, a one percent deviation in density of mass has a tiny effect on diffusion and chemical reactions, hardly noticeable. Diffusion forces are tiny perturbations on the electric force field and energies involved in diffusion are tiny perturbations in the total energy of charged systems. Electric forces and energies are not small parts of the total energy and it is not wise—to say the least—to treat them as perturbations of uncharged systems. Small changes in charge distribution produce large changes in flow because flow tends to be an exponential function of the spatial profile (even when the profile does not have a large, single, symmetrical peak [27] of potential energy and the profile of energy is a sensitive function of permanent and dielectric charge, ionic conditions, etc [38]. It is not wise [39] to neglect the effect of permanent charges of the channel protein on the shape and size of the electric field as is done in many continuum [11, 14-17, 20-22, 53, 54, 61, 96-105, 134, 145] and rate models of gating and permeation of ion channels, and most treatments of enzyme function and catalysis.

Charge is indeed an abstraction. In each of the devices in Fig. 2, charge follows different laws, because it has different physical properties, sometimes carried by charged particles, sometimes produced by the rate of change of electric field (displacement current), sometimes the 'movement' of quasi-particles, sometimes the 'movement' of electrons in a macroscopically delocalized quantum state of a wire.

Charge always flows without loss in each device. That abstract property of charge movement is always true, but the 'laws' describing current as a function(al) of time and potential (for example) depend on the physical nature of the charge and its movement. Different devices have different relationships between current, voltage, and time. These different 'constitutive' equations are described in the engineering literature in great detail.

The different constitutive equations combine with Kirchoff's current law to describe current flow from one place and one device to another. Together, the equations describe the universal fact that interrupting current flow in one chemical reaction (of a series of reactions) will interrupt current flow in every other reaction (in that series) even if the interruption is meters away from a chemical reaction being studied on the atomic scale (scale = 1 Å) and.

One is reminded of the power of the electric field when one unplugs a computer. Interruption of a circuit meters away from the diodes of the computer's power supply stops the flow of quasi-particles—holes and semi-electrons—across atomic scale junctions of semiconductor diodes, often of the *PN* variety. The electric potential changes so strongly in response to the interruption (because the power input of the computer power supply stores a great deal of charge) that the electric field exceeds the dielectric strength of air. Electrons are stripped off the atoms of the air, a plasma is created, a spark. We should be frightened of sparks and their electric field. Sparks start fires, sometimes in upholstery or drapes.

Vacuum tubes. In the next device in Fig. 2, we consider charge movement in a vacuum tube diode. Vacuum tubes control the flow of electron current by changes in their internal electric fields and were called 'valves' in the UK for that reason. Vacuum tubes, semiconductors and even some open ion channels follow simple constitutive laws of rectifiers, as described in textbooks of electronic devices or ion channels.

In vacuum tubes, current is indeed carried by a stream of isolated charged particles, electrons with a definite mass and charge, moving through a vacuum, interacting only through

their electric fields. At 1 Hz, essentially all the current in a vacuum diode is carried this way. Current through a diode is not proportional to the voltage across the diode because the electric fields within the diode change shape, despite the simple physics of conduction. The fields change shape as the voltage across the diode is changed because of screening and shielding. The electric fields within the tube are different at small and large potentials. The different internal electrical fields change the current flow, creating rectification. The electric field creates a large barrier in one direction so current in that direction is small; the electric field creates a small barrier in the other direction, and current in that other direction is large.

Rectification is of historical interest, because vacuum tubes allowed the early detection of radio waves in American's homes in the 1920's, as valves did in the UK. The demand for portable radios led to solid state diodes, then solid state 'triodes', field effect transistors, integrated circuits, and our modern digital world [132].

Vacuum tube diodes had certain difficulties—they were big, 10^{-2} m at their smallest, hot, greedy consumers of power, and electrically unstable: they drifted continuously. They were quickly replaced with semiconductor diodes, the next device in Fig. 2 that can be as small as 10^{-8} meters nowadays, costing $< 10^{-10}$ dollars each.

We turn now to a very different diode on a different scale. The electric field of charge must be respected on all scales—10^{-2} meters of vacuum tubes, 10^{-9} meters of semiconductor diodes—because the electric field has such large effects on all scales.

Semiconductor diodes. Current in semiconductors is carried by quasi-particles [86], called holes and 'electrons' (better named quasi-electrons, or semi-electrons in my view) These quasi-electrons and holes are defined because they interact much more simply than the totality of real electrons and lattice of atoms in the semiconductor [86, 126, 128, 144, 149]. They simplify the quantum mechanical many-body problem into the much simpler motion of imagined quasiparticles. Current is carried in semiconductors by rearrangements of charge in the entire lattice of atoms that make up the semiconductor. Fortunately for our technology and daily life, current in germanium and silicon can be approximated by classical theories that deal with one quasi-electron or one quasi-hole at a time as they move in mean fields, without requiring solution of the Schrödinger equation at all [78, 111, 115, 142, 149].

As a textbook puts it eloquently (p. 68 of [84]): "Electron is a quasi-particle consisting of a real electron and an exchange correlation hole … a cloud of effective charge of opposite sign due to exchange and correlation effects arising from interactions with **all** other electrons of the system." "Hole is a quasi-particle, like the electron, but of opposite charge; it corresponds to the absence of an electron from a single particle state that lies just below the Fermi level." The motion of these quasi-particles are described by mean field models, evaluated both by simulations [78, 149] and theory [78, 111, 115, 142, 149]. For example, the Poisson drift diffusion equations [81], often called *PNP* (for Poisson Nernst Planck) in biophysics and nanotechnology [7, 9, 12, 13, 28, 36, 37, 69, 153], are most important in semiconductors, not biophysics or physical chemistry.

Early treatments of rectification in biological channels, then called ionic conductances, ([53]; Appendix of [66]) in fact drew heavily on (nearly) contemporaneous treatments of rectification in diodes [121], but none of these were consistent: they assumed the electric field, instead of computing it, so the charge in the system did not produce the assumed electric field if substituted into Poisson's equation. Goldman [53], then a graduate student of K.C. Cole, recognized the problem, but did not know how to remedy it, nor did these workers [53, 66]

understand that consistent treatments produced electric fields that varied greatly with conditions, and were not constant in any sense, including space (numerous personal communications to me from K.C. Cole, 1960-1962, and A.L. Hodgkin, 1960-1995). The importance of computing the fields was not realized (as far as I know) until much later [36-38]. Until then, and even after, Nernst Planck equations were used in biophysics without including permanent charge of the protein [11, 14-17, 20-22, 54, 96-105, 134, 145] and without discussion of the change of shape of the electric field with concentrations, amino acid composition of the channel protein, and so on.

Charge carriers in semiconductors can be really quite strange from a physical point of view. Charge carriers in silicon and germanium do not exist outside the lattice of the semiconductor as distinct entities. They do not exist in the same sense that Na^+ and Cl^- ions exist, but are mathematical representations, with lifetimes sometimes as short as milliseconds. Quasi-particles are the second derivative of a Fermi surface of silicon and germanium semiconductors, under particular conditions. They are defined, as mentioned above, to allow easy classical analysis compared to the intractable quantum mechanical many-body problem of a macroscopic semiconductor.

Much of the success of our semiconductor, digital, and video technology is due to the accuracy of the constitutive *PNP* equations describing holes and semi-electrons. They robustly describe the characteristics of semiconductor devices of many different types, with very different input output relations, as different as an exponentiator and a logarithmic convertor.

PNP is so useful because almost all the devices of our digital technology work in a restricted set of conditions in which flows are crucial but are of the special type well described by quasi-particles moving in a mean field. Treatment of distortions of the electric field of all the atoms in the macroscopic device is not needed.

Almost all of the devices of electronics use power supplies to maintain different voltages at different locations far from the *PN* junctions of the device itself (and thus require a global treatment of the electric field). These voltages perturb the distribution of velocities of charged particles so the distribution has net flow [27, 40]. The slight perturbation is enough to imply the *PNP* equations and those provide enough nonlinearity to make amplifiers, switches, and the full set of logical circuits necessary to make a computer [126, 128, 144, 149]. In fact, in this case the *PNP* equations can be solved analytically and exactly to give intuitive pleasingly simple formulas for current flow, once the shape of the electric field is known [27].

PNP equations describe a wide variety of current voltage relations and devices. The *PNP* equations are the constitutive laws used to describe semiconductor devices as their current voltage relations change drastically (with voltage, for example) from that of a linear amplifier, to a switch, to an exponentiator, a multiplier or even a logarithmic converter. [126, 128, 144, 149] Nonlinear input output relations as varied as these enable a rich variety of devices.

Nonlinear input output relations as diverse as these are not described easily—or described at all, for that matter—in most areas of physics and chemistry. All the nonlinear devices in a computer are actually mathematical solutions of the *PNP* equations in a complex silicon structure built to have the particular spatial distribution of permanent charge ('doping') that produces the desired properties of the device, i.e., the input output relations.

The predictive power of *PNP* is very important in the design of robust semiconductor devices—that do not fail even when used many millions of times a second in computers that contain a trillion transistors. In fact, the intrinsic physical properties of semiconductors are adjusted by their designers so *PNP* remains a good description [18, 23, 24, 137], even as the size of the device has been

dramatically decreased. The concentration of fixed charge dopants, geometry, and recently even dielectric coefficients are adjusted by semiconductor engineers in their successive iterations of Moore's law [112, 118, 119], so *PNP* remains a good description as performance is increased by factors of a billion or so, over 50 years.

Evidently, reliable design is more important than raw performance. It seems more important for the designers (and marketplace) that an equation describes behavior robustly and accurately over a range of conditions than that the device be as fast or small as possible [18, 23, 24, 137]. Evolutionary selection in biology also seems to choose robustness over efficiency in many cases.

Devices of nanotechnology need to be similarly robust, I believe, before they will be used extensively.

PNP is not enough, however, when ionic solutions are involved. A great deal of effort has been spent applying *PNP* equations to electrochemical systems [7, 9, 12, 13, 28, 36, 37, 69, 153] hoping they might serve as adequate robust constitutive equations, but that is not the case. The nonideality of ionic solutions, arising in large measure from saturation effects produced by the finite size of ions, demands more powerful mathematics than the partial differential equations of *PNP* used in computational electronics.

Resistor. The next device we discuss in Fig. 2 is a resistor, which in some ways is the easiest to describe, because current is proportional to voltage with a single proportionality constant over a wide range of voltages, times, and conditions. A resistor follows Ohm's constitutive law with a resistance independent of potential over a wide range from 10^{-5} volts to say 100 volts, and from values of resistance from 10^{-1} ohm to 10^8 or 10^9 ohms.

The range of validity of Ohm's law is an enormous help in circuit design. Circuit models involving resistors, capacitors, inductors and operational amplifiers are transferrable. They behave as real devices behave without changes in parameters. Largely for that reason, designs are inexpensive and robust.

Despite the simplicity of resistors, the actual current carrier in a carbon resistor, is unclear, at least to me. No one cares very much I suspect, because the device works nearly perfectly. The carrier of charge does not matter very much. What matters is the constitutive law that describes the relation of current, voltage and time. The constitutive law should satisfy conservation of mass, charge and current.

It is instructive to write the constitutive law for a resistor Ohm's law for only particle current, using a conservation of particle (mass) formulation, and then write it again for particle plus displacement current from one terminal to another. If you apply a step function of current (or potential for that matter), to the purely particle formulation, a paradox arises. The potential changes but the particle current flowing into the resistor from the left is exactly equal to the particle current flowing out on the right at all times. Why does the potential change if there is no accumulation of charge?

The paradox can be resolved in two (nearly) equivalent ways.

(1) The constitutive equation Ohm's law can be used with the extended definition of current that includes the displacement current. In that case there is continuity of generalized current, but there is *NOT* continuity of particle current at all times. The transient accumulation of particle current provides the charge that changes the potential. Paradox resolved.

(2) Alternatively, the circuit model of the resistor in Fig. 2 can be changed to have an explicit capacitor in parallel with it. In that case, the charge accumulates on the capacitor, and the resistor itself can have continuity of flux of particles at all times and follow Ohm's law using the current/flux of particles (and not the displacement current). The charge accumulating on the capacitor creates and changes the electric field. Paradox resolved.

Maxwell himself repeatedly used capacitors in this spirit to understand the role and significance of displacement current. Sections 102, 125, 199, and Chapters 8, are some examples, in [116]. We (following the insight and advice of Wolfgang Nonner) have used capacitors as a crude way to connect permanent charges (specifically, acid side chains in a channel protein away from the pore) and electrical potential in the pore of a channel protein, e.g., the potassium channel [45].

Current flow in wires. The current carriers in a wire are delocalized electrons in the simple case of a single solid conductor of metal, and follow the simplest constitutive law of all at long(ish) times, say times longer than 10^{-5} sec. But most of our electronics occurs at times much shorter than that.

We confront the importance of the time variable. The physical nature of current flow depends on time scales, even in wires [83, 139]. The range of time scales in our technology is enormous, from more than one second to less than 10^{-9} sec. At the shorter times $(< 10^{-4}$ sec$)$, the wires that must be used are often twisted pairs [8], each made itself of many very fine wires. Without twisting, these pairs of wires do not allow successful connections to the internet because rapidly changing signals are not carried reliably by single wires [146]. At those short times, currents flow outside wires, guided by the conductor, to be sure, but outside the conductor nonetheless [83, 146]. The twisting of wires is a necessity if they are to carry signals robustly and reliably so we can use them in our video devices and smartphones, even in old fashioned hard wired telephones [8] that only need to amplify audio signals heard by adult humans $< 10^4$ Hz.

Physical nature of current depends on time scale. The physical nature of current in almost any system depends on the time scale, and differs at different times as much as it differs in different devices. Constitutive equations depend on time. Different devices have different constitutive equations with different time dependence.

Again, charge movement is an abstraction, different at different times in one device, as well as different in different devices.

Charge Movement in Batteries. Batteries are present in Fig. 2 both as an isolated device and as the $Ag\|Ag^+Cl^-$ interface between wires and Na^+Cl^- solution in the conducting cylinder previously discussed.

I hesitate to describe a constitutive equation for the flow of charge in batteries in general because the flow is so very complex, different in different devices, and important for the practical daily use of batteries [25, 85] and its interaction with surface charges is also subtle and important [138]. It is enough to say here that current flow through electrochemical systems is carried by a wide range of charge carriers. Constitutive equations of different electrochemical systems differ and change nature dramatically with time and frequency and composition and concentration of ionic solutions, as well as electrical potential, and current flow. At short times, at 10^{-6} sec—that are still long and slow compared to the times important in computers—current from the Ag wire and AgCl electrode material into the Na^+Cl^- solution is entirely displacement current lagging behind voltage but at the longer times characteristic of biological systems

(greater than say 0.1 sec) the current is carried by a complex combination of Ag^+ and Cl^- ions with negligible displacement current.

We conclude that charge is indeed an abstraction with different physical meanings in different systems and at different times. No one can visualize and no one knows—at least I do not know anyone who knows—why or even how this abstraction can be so perfectly conserved under all conditions and on all scales, from Angstroms to meters, from femtoseconds to seconds. The reader may have difficulty visualizing the interactions that enforce conservation of the abstraction 'charge/current' in all these devices with all these properties over the entire time scale. I certainly do. However,

Charge is conserved on and between all scales in all conditions, even if we cannot visualize how that manages to be so. Experiments demonstrate that fact. Current does flow continuously without loss in a circuit. Consider a battery feeding a circuit. If a wire is cut far from the battery, current flow stops everywhere. The chemical reaction in the battery is disrupted on an atomic scale, by the (lack of) current flow meters away. Those of us living in colder climates have seen the effects when starting a car with another car's battery. We have learned to be careful because even a twelve volt battery can produce dangerous sparks in air, even though air has nearly infinite resistance, and is in that way, nearly a vacuum. Abstract charge is conserved 'exactly' even if we cannot visualize how that happens.

Science often contains mysteries that cannot be visualized—consider Maxwell's attempts to visualize his equations as properties of an ether. Science often poses questions that cannot be answered. Why is there no magnetic charge? Why is the charge on an electron 1.6×10^{-19} coulombs? Why are physical laws invariant when locations s move at constant velocity $\partial s/\partial t$—special relativity—or at constant acceleration $\partial^2 s/\partial t^2$—general relativity—but not when other time derivatives of location are constant, like a constant third derivative $\partial^3 s/\partial t^3$ or linear combinations of $\partial^n s/\partial t^n$? As practical people, scientists cannot afford to just wait while we wonder about such things. Scientists wonder a bit **and** then move on, hoping our successors can do better than we have.

Biologists and engineers in particular cannot afford to linger on mysteries they do not understand. So many of those mysteries in biology have turned out to be caused by low resolution of our instruments, unable to resolve crucial structures. Think of Thomas Henry Huxley looking at the shortening of the striations of muscle [75] that were not understood until one of his grandsons (Andrew Huxley), studied them many years later working in Cambridge UK near an unrelated Huxley (Hugh) [72]. Think of Lee DeForest using vacuum tubes without understanding how they work.

Biologists and engineers cannot afford to wait to understand everything. They must isolate the mysteries and move on to study other things. Here, we move on to discuss devices and the theories and simulations used to understand them.

Biological Implications of Kirchoff's Current Law. It is important to note that the Kirchoff's current law has important biological implications in systems more general than a series of chemical reactions. Kirchoff's current law implies the cable equations (called the telegrapher's equation in the mathematics literature). The cable equation [65] is the foundation of the Hodgkin Huxley model [64, 67, 71, 73, 74] of the action potential of nerve and muscle fibers. Kirchoff's current law links the atomic properties of ions, the molecular properties of ion channels, and the

centimeter scale spread of current and potential that creates the propagating action potential in nerve fibers meters in length.

In short or round(ish) cells, Kirchoff's current law forces coupling between multiple pathways of current crossing membranes, even if the currents are carried by different ions, or by electrons, through different structures nanometers apart in the membrane of the finite cells or organelles.

The flux coupling characteristic of active transport systems—including the coupled flows in chemi-osmotic systems that perform oxidative phosphorylation or photosynthesis—might arise in this manner. Coupling of flows of charges, whether electrons or ions, is an unavoidable consequence of GLOBAL conservation of charge and current, of Kirchoff's current law GLOBALLY enforced, and not a consequence of local chemical interactions, just as coupling of membrane currents with axial currents in a nerve fiber is an unavoidable consequence of Kirchoff's current law, not of local chemical reactions.

It is interesting to compare the incorrect chemical theory of nerve propagation of Nobel Laureate A.V. Hill [60] with the correct electrical theory of the then undergraduate [62, 63], later Nobel Laureate [64, 67, 71, 73, 74] Alan Hodgkin. Kirchoff's current law in the form of the cable equation [65] was the key to Hodgkin's understanding. The classical voltage clamp experiments were designed to remove difficult terms, and isolate membrane terms, in the cable equation—personal communication, A.L. Hodgkin, 1961—that today we know describe ion channels, opening, closing and conducting.

Cable equation links movement of atoms inside channel proteins to macroscopic current flow that produces nerve propagation of the macroscopic electrical potential, the nerve signal that spreads meters. Macroscopic potentials modify atomic movements involved in gating and conduction. Atomic movements create the macroscopic electrical potentials.

Equations of the electric field are true on all scales and so allow a unique linkage between models of atomic motion, protein behavior, and macroscopic propagation of electrical signals. I suspect linkage equations of this type—valid on all scales—will be needed to make any multiscale analysis robust and transferrable, if it reaches from atoms to meters, from femtoseconds to minutes as models of nanodevices must.

Models, Devices, Effective Parameters, and Transferrable Theories. Parameters of models or devices can often be chosen so an incomplete theory or simulation describes a system in one set of conditions. Chemistry and biology are filled with examples. Chemical reactions follow rate equations, but the rates are not constant, not independent of one another, as conditions change, even though theory assumes they should be [33, 34]. Biology describes enzymes with one set of parameters but finds those are changed when conditions change and attributes that somewhat mysteriously to 'allosteric effects' and conformation changes.

Biology and much of chemistry works under a wide range of conditions and so incomplete theories with effective parameters have limited use. Even if sensible, even if valid, theories (and simulations) with effective parameters like these are not accurate enough to design robust devices. By leaving out something important, those theories or simulations leave out an energy term that is almost certain to vary with conditions. The resulting effective parameters change in large and unpredictable ways.

Incomplete theories and simulations are not very useful over a range of experiments and conditions. Incomplete theories are not likely to be transferable (from one set of conditions to

another) in the language of the chemistry literature. Devices designed from incomplete theories or simulations are unlikely to be robust or work well under a range of conditions.

Biological systems analyzed with non-transferable theories (or simulations) are unlikely to be realistic in general because biological systems almost always work in a range of ionic concentrations different from those used in the laboratory.

Biological systems usually work in mixtures with a range of Ca^{2+} concentrations, in which Ca^{2+} concentration has important practical effects, often turning systems on or off or controlling their rate monotonically. Simulations and theories in biology have limited use until they are calibrated so we can be sure they actually are correct in the range of conditions and Ca^{2+} concentrations the biological system uses. Simulating Ca^{2+} activity in pure solutions is a challenging problem [140]. Simulating Ca^{2+} activity in biological mixtures in the 10^{-8} to 2×10^{1} molar concentrations that are physiological, has not been attempted as far as I know. (10^{-8} concentrations of Ca^{2+} are found inside most cells. 2×10^{1} concentrations of Ca^{2+} are found in and near ion channels, nucleic acids, and enzyme active sites, where the chemistry of life is catalyzed and controlled.

Biological and chemical science will benefit enormously if theories and simulations can be made transferable, using one set of parameters to describe systems in a range of conditions, as many physical and most engineering theories and simulations do. I believe the law of mass action must be extended to conserve charge before theories and simulations can be made transferable from condition to condition, from physics, to chemistry to biology, using only mathematics.

How to extend the law of mass action? An obvious way to extend the law of mass action is to include an electrical potential in the rate constant to 'right the rates' by making them exponential functions of potential. This in fact has been done for a very long time in the study of reactions at the electrodes of electrochemical cells. The Butler-Volmer and Tafel equations [3] include electrical potential in rate constants in an empirical way with limited [55, 56] but real success.

Success is limited I suspect because difficulties of just embedding an electrical potential in a rate constant are formidable. We must 'fix the fields' so they are global and depend on current flow everywhere. The only way to 'right the rates' is to fix the fields, everywhere. Otherwise, they cannot conserve charge and support continuity of current as required by Kirchoff's current law.

(1) The rate constants in one reaction scheme must depend on the electrical potential in far locations. Otherwise, interrupting current flow in a far location is not likely to interrupt current locally. This requirement implies that the electrical potential must be determined by a global equation like Poisson's equation, including boundary conditions far from the individual chemical reactions.

(2) The rate constants in one reaction scheme are likely to depend on concentrations throughout the system because the solutions containing the reactants are not ideal. A general characteristic of nonideal solutions is that 'everything depends on everything else'. More specifically, the activity of one reactant (the free energy per mole) depends on the concentrations of other species in practice, as well as in principle.

(3) A general theory of all nonequilibrium processes is not likely to be useful: a general theory has to describe too much. A general theory must include hydrodynamic behavior of considerable complexity, since aqueous solutions are fluids satisfying the Navier Stokes equations of fluid mechanics. A general theory would also include

explosions since they occur with regrettable frequency at electrodes of electrochemical cells, when H_2 gas is generated (inadvertently) by an overvoltage.

(4) A thermodynamic treatment is clearly impossible since the goal is to calculate large fluxes and currents that do not occur in a thermodynamic system at equilibrium by definition without flows. Clearly a frictional treatment of flux over a large potential barrier as in the classical drunken man problem of Kramers [44, 57] is a necessary step forward, even if it is inconsistent because it does not compute the potential barrier from the charges in the system. A simple generalization of Kramers for any shape barrier is available [27] and needed [2] because so few barriers are both symmetrical and large as required in classical high barrier approximations.

<u>Variational methods can extend the law of mass action</u>. If the goal is to build transferable systems, so we can build robust devices, as in electronic technology, we must use a mathematics that allows interactions of charges and fields, extending from atomic to macroscopic scales.

Variational methods are designed to deal with systems with multiple forces and flows, in which interactions are unavoidable and complex. In these systems, interactions must be included in all analysis. Otherwise, theories have more adjustable parameters than can be determined experimentally and still cannot deal with a range of conditions, because interactions change with conditions in ways too complex for ordinary theories.

Sadly, theories of ionic solutions—even the assignment of properties to single ions in infinitely dilute solutions [70]—seem to have these difficulties. Theories of ionic solutions need a large number of adjustable parameters and still cannot describe biological solutions (for example). References [29, 32, 47, 48, 87-89, 92, 127, 143, 157] draw particular attention to the difficulties.

The sad limitations of our understanding of ionic mixtures, like those in which all of biology occurs, are not widely known, and so embarrassing that many do not want to know of it, as I did not for many years. It is necessary then to document the frustration by quotations from leading workers in that field.

The classical text of Robinson and Stokes [133] is still in print and widely used. It is a book not noted for emotion that still gives a glimpse of its authors' feelings of frustration (p. 302)

> "In regard to concentrated solutions, many workers adopt a counsel of
> despair, confining their interest to concentrations below about 0.02 M,
> ... "
> *(Almost all biology occurs in much more concentrated solutions.)*

In a recent comprehensive treatment [89] of nonideal properties of solutions, the editor Werner Kunz says (p. 11 of [90])

> "It is still a fact that over the last decades, it was easier to fly to the
> moon than to describe the free energy of even the simplest salt
> solutions beyond a concentration of 0.1M or so."

New mathematical tools are needed in my view to remedy this situation which has been stalemated since the 1920's. The powerful tools of variational calculus automatically deal with interactions that vary dramatically with conditions. If the mathematics does not deal with

interactions, those interactions will not be computed correctly and will wreck theories that are based on algebraic descriptions of interactions or theories based on the law of mass action with constant rate constants.

Energetic variational methods are particularly useful because they allow multiscale derivation of partial differential equations (and far field boundary conditions) from physical principles when multiple fields are involved, like convection, diffusion, steric exclusion, and migration in an electric field. Energetic variational principles have recently become available for systems involving friction [46, 76, 154], that is to say, for systems involving ionic solutions [35, 69, 120, 152].

Energetic variational principles combine the full power of the Navier Stokes equations (1) with either a Lennard Jones representation of finite size ions or (2) with a density functional theory of ionic solutions built from Rosenfeld's density functional theory of liquids [35, 77].

Computations must be done in three dimensions because spheres do not exist in one and two dimensions. These theories and their simplifications [68, 93, 107] are difficult to compute in three dimensions because of the steeply singular forces used to ensure that atoms do not overlap. Overlap must not be permitted because spheres cannot overfill space: space can be saturated with spheres. Saturation effects are a main cause of nonideality, particularly in the extremely crowded conditions in and near enzyme active sites, ionic channels, nucleic acids, and the working electrodes of electrochemical cells where twenty molar solutions are not uncommon [82]. (As a rule of thumb, ions are crowded and electric fields are largest [50, 59, 147] where they are most important in technology and biology.)

If saturation is described by a Fermi-like distribution—as recently derived for spheres of unequal sizes [108, 109]—some of these difficulties can be attacked. A fourth order partial differential equation can be written [110] which is easily integrated in three dimensions, after it is reduced to a pair of second order partial differential equations (with carefully defined boundary conditions) and computed with appropriate numerical methods.

But it is still not clear how best to apply any of these methods to chemical reactions (Fig. 1) described by the law of mass action with rate constants extended to be functions or functionals and not constants.

And that is not a bad place to move this essay towards its coda. We now have the tools, and we now see the goal—a global treatment combining conservation of mass in chemical reactions described by the law of mass action with conservation of charge and current described by Kirchoff's current law everywhere. Now we have to do the work.

Coda: our grasp must be sure, but our reach should exceed our grasp, as we do the work. We must grasp both charge/current conservation and mass action before we can produce robust theories (or simulations) of chemical reactions in ionic solutions that successfully use one set of parameters in a range of conditions, and include the global properties of the electric field.

Correct calculations are needed because there is no engineering without numbers and accurate computations [26, 114, 129]. Calculations from theories and simulations—of electronic or ionic or biologically inspired devices—must be checked and calibrated against known results. Otherwise devices built from those calculations will not work. There is little engineering without robust theories because devices have much less tolerance for error than abstractions. If theories and simulations of electrical devices were not robust, if parameters had to be changed as conditions changed, our electronic technology would be severely limited, to say the least.

Chemical reactions involving current flow must be within our theoretical grasp before we can develop transferable theories. Only then can we build devices that perform as expected, as electronic devices usually do, in my opinion. Only then can we expect exponential growth in molecular engineering whether biological or technological. It seems no coincidence that exponential growth in electronic technology came after scientists had a secure grasp of global electrostatics and the *PNP* equations of electrodiffusion in semiconductors.

Let us hope that energetic variational methods can grasp ions and chemical reactions in water as well and as successfully as *PNP* has grasped the useful properties of holes and (semi)-electrons in silicon and germanium.

Acknowledgement

Appendix
When does mass action conserve charge?
What does unconserved charge do?

Size of discontinuity of current flow. The reader may hope that the amount of charge involved in the discontinuity of current flow is too small to matter, but small charges have large effects because of the strength of the electric field.

The difference in current shown in eq. (9) is the discontinuity of current, the violation of Kirchoff's law of continuity of current flow.

$$\boxed{\frac{I_{XY} - I_{YZ}}{F} = z_X \cdot k_{xy}[X] - z_Y \cdot k_{yx}[Y] - z_Y \cdot k_{yz}[Y] + z_Z \cdot k_{zy}[Z]} \qquad (9)$$

Special Cases when both conservation laws are satisfied. When $I_{XY} - I_{YZ} = 0$ in eq.(9) –eq. (12), the law of mass action is consistent with Kirchoff's current law and conservation of mass and conservation of charge/current are all satisfied.

Significance of the error is seen by considering reduced cases, remembering that the units of current $I_{XY} - I_{YZ}$ here are $(\text{cou/sec})/\text{liter}$.

Special Case A: If all **concentrations are set equal to one**, the currents (in the special case with a tilde)

$$\frac{\tilde{I}_{XY} - \tilde{I}_{YZ}}{F \cdot 1\frac{\text{mole}}{\text{liter}}} = z_X \cdot k_{xy} - z_Y \cdot k_{yx} - z_Y \cdot k_{yz} + z_Z \cdot k_{zy}; \quad \text{concentrations} = 1\frac{\text{mole}}{\text{liter}} \qquad (10)$$

Special Case A*: If we also set all **charges equal to one**, along with **concentrations equal to one**,

$$\frac{\hat{I}_{XY} - \hat{I}_{YZ}}{F \cdot 1\frac{\text{mole}}{\text{liter}}} = k_{xy} - k_{yx} - k_{yz} + k_{zy}; \text{concentrations} = 1\frac{\text{mole}}{\text{liter}}; z_X = z_y = z_Z = 1 \qquad (11)$$

In this special case of eq. (11), labelled **A***, asymmetry (net difference) in rate constants determines the discontinuity of current, the violation of Kirchoff's current law.

Special Case B: Alternatively, we can set all **rate constants** and all **concentrations equal to one**,

$$\frac{\hat{I}_{XY} - \hat{I}_{YZ}}{F \cdot 1\frac{\text{mole}}{\text{liter}}\frac{1}{\text{sec}}} = z_X - z_Y - z_Y + z_Z; \text{concentrations} = 1\frac{\text{mole}}{\text{liter}}; \text{rate constants} = 1\frac{1}{\text{sec}} \qquad (12)$$

In this special case of eq. (12), labelled **B**, asymmetry (net difference) of charges (valences) determines the discontinuity of current, the violation of Kirchoff's current law.

<u>Asymmetry of parameters violates conservation of charge/current</u> and produces discontinuity in current from device to device, i.e., it produces accumulation of charge, with sizable effects, as shown next.

<u>Size of effects.</u> To estimate the effect on electrical potential *V,* we need to know the size of the system. Imagine a spherical capacitor of radius *R*. Its capacitance to ground—or coefficient of the self-energy, if one prefers the phrase in the chemical literature—is $C = Q/V = 4\pi\varepsilon_r\varepsilon_0 R$ or numerically $4\pi \times 8.85 \times 10^{-12}$[farad/meter]$\varepsilon_r R = 1.11 \times 10^{-10} \varepsilon_r R$ [farad] where ε_r is the relative dielectric coefficient, about 80 in water solutions at longish times (say $> 10^{-5}$ sec). Then, a 1 nm radius capacitor with dielectric coefficient 80 has capacitance of 8.9×10^{-18} farads. Small charges produce large voltages in such a tiny capacitor. Even the charge on just one ion $(1.6 \times 10^{-19}$ cou) would produce 18 mV, large enough (compared to the thermal potential of 25 mV) to have a noticeable (~50%) effect in theories and simulations, because $\exp(-18/25) = 0.49$. (Rates often vary exponentially according to $\exp(-V/k_BT)$.) A unit discontinuity in current $I_{XY} - I_{YZ}$ in eq. (10) − (12) lasting for a second would produce a voltage of $V = Q/C = (1/F)/(8.9 \times 10^{-18}) = 1.14 \times 10^{12}$ volts.

Of course, 1 second is a long time for current to flow. If current flowed on a biological time scale, for 1 msec in a structure 1 nm in radius, with dielectric coefficient 80, the electrical potential would be much less, 'only' $10^9/\varepsilon_r$ volts. Current flow of even a microsecond, would produce some thousands of volts.

<u>Rate of change of potential</u>. We can also look at the effect on the rate of change of potential. The discontinuity of current is connected to the rate of change of potential by a version of Coulomb's law

$$\frac{\partial V}{\partial t} = \frac{1}{C} I \tag{13}$$

If we apply this formula to the discontinuity of current in the special case of eq. (11), labelled *A**, we can estimate how quickly that discontinuity of current would change the potential

$$\frac{\partial V}{\partial t} = \frac{\hat{I}_{XY} - \hat{I}_{YZ}}{F \cdot 1 \text{(mole/liter)}} \frac{F}{C} = \frac{\hat{I}_{XY} - \hat{I}_{YZ}}{F \cdot 1 \text{(mole/liter)}} \frac{F}{4\pi\varepsilon_r\varepsilon_0 R} = F \frac{k_{xy} - k_{yx} - k_{yz} + k_{zy}}{4\pi\varepsilon_r\varepsilon_0 R} \tag{14}$$

For a capacitor $R = 1$ nm with $\varepsilon_r = 80$ and capacitance of 8.9×10^{-18} farads (see above)

$$\frac{\partial V}{\partial t} = 1.1 \times 10^{17} \text{farads} \times F\left(k_{xy} - k_{yx} - k_{yz} + k_{zy}\right) \quad \text{in volts/sec} \tag{15}$$

In other words, the breakdown voltage (~ 0.2 volts) of membranes and proteins would be reached in $1.82 \times 10^{-18}/F\left(k_{xy} - k_{yx} - k_{yz} + k_{zy}\right)$ sec, not a very long time. The breakdown voltage for matter (say 10^6 volts) would be reached very very quickly.

We conclude that

failure of the law of mass action to conserve charge is likely to have noticeable effects.

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