Maxwell and Conservation of Current

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$$\boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t \quad \Rightarrow \quad \nabla \cdot \left(\mathbf{J} + \varepsilon_0 \partial \mathbf{E}/\partial t \right) = 0, \text{ no matter what } \mathbf{J} \text{ is!}}$$

It is remarkable that scientists do not today share the wonder at Maxwell's equations that he, Einstein, and the geniuses who created Quantum Mechanics shared.

A glance at the sun proves that light propagates enormous distances according to Maxwell. (The stars of course show the same over much larger distances.) The only way propagating light can emerge from the electric field is if a time rate of change of the electric field IN EMPTY SPACE creates a magnetic field, i.e., Ampere's law must be generalized to include a $\varepsilon_0 \partial \mathbf{E}/\partial t$ term.

That term is present everywhere, because atoms are mostly empty space.

The properties of light propagating in matter similarly require the existence of a $\varepsilon_0 \partial \mathbf{E}/\partial t$ term.

'light' here stands in for any electromagnetic wave of course, including x-rays and gamma rays of much higher frequency than light itself.

Thus, conservation of current (when current includes the $\varepsilon_0 \partial \mathbf{E}/\partial t$ term) is universal and must be obeyed in atomic scale simulations, chemical models, etc etc.

Of course, this agrees with the experience of anyone who has handled electricity in the form of a circuit. Circuits are one dimensional structures in which nothing happens unless current is allowed to flow everywhere, i.e. current flows in completed circuits as any electrical engineer learns right away.

These are lots of words to say

$$\boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t \quad \Rightarrow \quad \nabla \cdot \left(\mathbf{J} + \varepsilon_0 \partial \mathbf{E}/\partial t \right) = 0, \text{ no matter what } \mathbf{J} \text{ is!}}$$

Here J can be any physically possible function at all (I do not know what happens when bizarre discontinuities etc are allowed) and SPECIFICALLY it includes what is called induced or polarization or material dielectric current, namely the current in matter proportional to $\varepsilon_0 \partial \mathbf{E}/\partial t$ even if the proportionality is very time dependent and spatially complex.

The $\varepsilon_0 \partial \mathbf{E}/\partial t$ term varies to make $\nabla \cdot \left(\mathbf{J} + \varepsilon_0 \partial \mathbf{E}/\partial t \right) = 0$ no matter what are the properties of matter, i.e., no matter what are the properties of $\mathbf{J}$. In fact, it is a common occurrence that $\mathbf{E}$ can become so large that the PHYSICS of $\mathbf{J}$ is changed. For example, when you pull a plug from the wall, $\partial \mathbf{E}/\partial t$ and then $\mathbf{E}(t)$ becomes so large that the atoms of air are stripped of their electrons and become a plasma, with physics as far from that of previously insulating air as one could imagine.