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Well posedness

Bob Eisenberg <beisenbe@rush.edu> **Mon, Feb 6, 2012 at 7:15 AM**
Reply-To: beisenbe@rush.edu
To: Xiaofan Li <lix@iit.edu>, Allen Flavell <aflavell@hawk.iit.edu>, Chun Liu <liu@math.psu.edu>, Bob Eisenberg <beisenbe@rush.edu>

Dear Xiaofan

Sorry I could not make it today.
One good point about meeting next
week is that we have time to "talk"
about the well posedness stuff.

The basic problem can be seen two
different ways.

ONE WAY motivated by experiments

1) In experiments the parallel capacitance is
ALWAYS present and it is known to be impossible
to do an experiment if it is reduced to too low a value
(instability results).

2) In experiments the parallel condutance is ALWAYS
present. It is removed by data analysis before the results
are published ('leak subtraction' or something like that is
the name).

SECOND WAY motivated by mathematics. This I will state
not as well. I depend on you and Chun to correct my mistakes.

1) Consider very long times. If one applies the divergence theorem

(i.e., integrate by parts over the entire domain) one isolates the fluxes. These must balance to zero over infinite time if there is to be a stationary infinite time solution (i.e., concentrations are stationary). This cannot be done BOTH for all fluxes and ALSO for the electrical current in the Nernst Planck formulation itself. There are N+1 conditions and N variables. (N fluxes for N ion types plus one for current). Because of scaling, the current boundary condition MUST ALWAYS be enforced. (Small error in current makes BIG effect on fluxes. OR to say the same thing the other way. Setting the fluxes all to zero does not imply zero current within reasonable numerical accuracy.)

SO we have to change the problem by adding an additional flux. We do this by introducing the leak path in parallel with the Nernst Planck fluxes. We choose the properties of this path to perturb the normal time (NOT going to infinity) as little as possible

2) If there is no flux in the problem, or if we look at VERY short times, the NP equations produce massive trouble (I will leave it to you and Chun to define this mathematically). But if we put in the parallel capacitance, these problems disappear. The charge on this capacitor is always EXACTLY the charge that would cause the problems otherwise. Or in electrical language, capacitive current through the capacitor (which is NOT carried by any ions but rather by the elecric field itself) is equal to the sum of the ionic fluxes in current units.

3) putting capacitor and shunt leakage conductance into the problem solves everything.

It is also what the experimenters actually do.

Let me know if this makes sense.

As ever
Bob

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