

# Conservation of Current is Universal and Exact *in five slides*

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## Maxwell's Magnetism

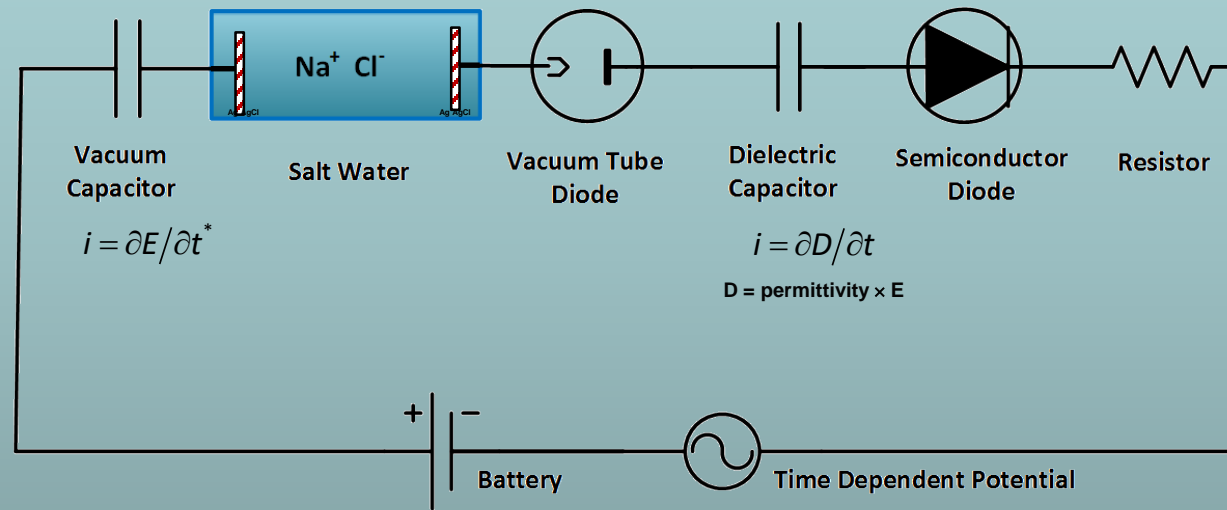
$$\mathbf{curl}(\mathbf{B}(x, t)/\mu_0) = \underbrace{\mathbf{J}(x, t) + \epsilon_0 \frac{\partial \mathbf{E}(x, t)}{\partial t}}_{\text{Current}}$$

## Current is Conserved **PERFECTLY**

$\mathbf{div curl} \equiv 0$  is an identity

$$\mathbf{div} \left( \underbrace{\mathbf{J}(x, t) + \epsilon_0 \frac{\partial \mathbf{E}(x, t)}{\partial t}}_{\text{Current}} \right) = 0$$

**'Charge' has  
Very different Physics  
in different systems**



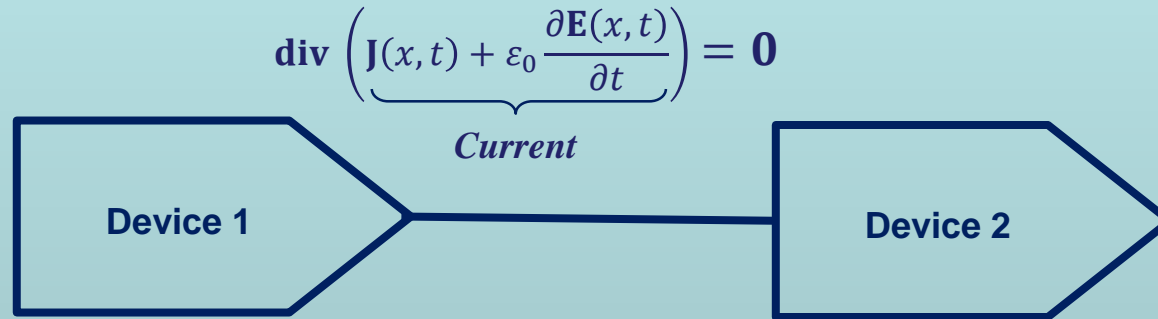
*but* **Continuity of Current is  
Exact**  
*No matter what carries the current!*

# Conservation of Current is Exact and Universal

$$\text{div} \left( \underbrace{\mathbf{J}(x, t) + \epsilon_0 \frac{\partial \mathbf{E}(x, t)}{\partial t}}_{\text{Current}} \right) = 0$$

even though  
Physics of Charge Flow  
Varies Profoundly

How can that possibly be?



**Displacement Current  
is  
Different in Each Device**

*because*

*$\mathbf{E}(x, t)$  is Different in every Device*

**so the**

**TOTAL Current is exactly equal**  
*at every time in every location and every device*

*Total Current = Displacement Current + Device Current*

# Electric Field takes on the Value that Conserves Current

$$\mathbf{E}(x, t) = -\frac{1}{\varepsilon_0} \int \mathbf{J}(x, t) dt$$

*Specifically,*

**E changes the displacement current  $\varepsilon_0 \partial \mathbf{E} / \partial t$**

*(that Maxwell called the polarization of the vacuum)*

So total current  $\mathbf{J}(x, t) + \varepsilon_0 \partial \mathbf{E} / \partial t$  is always conserved

Details and PROOF  
including quantum mechanics at  
<https://arxiv.org/abs/1609.09175>