Conservation of Current is Universal and Exact

in five slides

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Maxwell’s Magnetism

\[ \text{curl}(\mathbf{B}(x, t)/\mu_0) = \mathbf{J}(x, t) + \varepsilon_0 \frac{\partial \mathbf{E}(x, t)}{\partial t} \]

**Current is Conserved**

PERFECTLY

\[ \text{div} \text{ curl} \equiv 0 \text{ is an identity} \]

\[ \text{div} \left( \mathbf{J}(x, t) + \varepsilon_0 \frac{\partial \mathbf{E}(x, t)}{\partial t} \right) = 0 \]
‘Charge’ has Very different Physics in different systems

\[ i = \frac{\partial E}{\partial t} \]

\[ D = \text{permittivity} \times E \]

but Continuity of Current is Exact

No matter what carries the current!
Conservation of Current is Exact and Universal

\[
\text{div} \left( J(x, t) + \varepsilon_0 \frac{\partial E(x, t)}{\partial t} \right) = 0
\]

even though

Physics of Charge Flow
Varies Profoundly
Displacement Current is Different in Each Device because $E(x, t)$ is Different in every Device

so the

TOTAL Current is exactly equal

at every time in every location and every device

Total Current = Displacement Current + Device Current

\[
\text{div} \left( J(x,t) + \varepsilon_0 \frac{\partial E(x,t)}{\partial t} \right) = 0
\]
Electric Field takes on the Value that Conserves Current

\[ E(x, t) = -\frac{1}{\varepsilon_0} \int J(x, t) \, dt \]

Specifically, \( E \) changes the displacement current \( \varepsilon_0 \frac{\partial E}{\partial t} \)

(that Maxwell called the polarization of the vacuum)

So total current \( J(x, t) + \varepsilon_0 \frac{\partial E}{\partial t} \) is always conserved

Details and PROOF
including quantum mechanics at
https://arxiv.org/abs/1609.09175