

see <https://arxiv.org>, article 1901.10805

Dielectric Dilemma

Robert S. Eisenberg
Department of Applied Mathematics
Illinois Institute of Technology;
Department of Physiology and Biophysics
Rush University Medical Center
Chicago IL
Bob.Eisenberg@gmail.com

February 4, 2019

File name: Dielectric Dilemma February 3-2 header 2019.docx

Keywords

conservation of current, Maxwell equations,
electrodynamics, Kirchoff's law

Abstract

A dielectric dilemma faces scientists because Maxwell's equations are poor approximations as usually written, with a single dielectric constant. Maxwell's equations are then not accurate enough to be useful in many applications. The dilemma can be partially resolved by a rederivation of conservation of current, where current is defined now to include the 'polarization of the vacuum' $\epsilon_0 \partial \mathbf{E} / \partial t$. Conservation of current becomes Kirchoff's current law with this definition, in the one dimensional circuits of our electronic technology. With this definition, Kirchoff's laws are valid whenever Maxwell's equations are valid, explaining why those laws reliably describe circuits that switch in nanoseconds.

Dielectrics pose a dilemma for scientists. Dielectrics are not well described by the classical equations of electrodynamics, as they are usually written.

The equations of Maxwell [1-5] seem perfectly general in a vacuum, but, in the presence of dielectrics or matter, Maxwell's equations are not general at all. They describe polarization¹ of matter with a severe approximation. They describe polarization by single dielectric constant, a single real number ≥ 1 . This description is nearly universal in textbooks, and in the minds of students, but a single dielectric constant is too approximate to be useful in a technological world where computer circuits switch faster than 10^{-9} sec and in a biological world where electrical signals are controlled by atomic movements $< 10^{-10}$ m within proteins.

The significant variation of dielectric parameters with time/frequency and conditions has been known a long time [6, 7]. Dielectric parameters have been studied extensively since then [6-36] because polarization is a major determinant of the forces between molecules [37, 38]. The experimental results are remarkably diverse. In solids, dielectric properties depend on time/frequency in every material. In ionic solutions, dielectric properties depend—'without rhyme or reason'—on the type and concentrations of ions [11, 12]. Ionic solutions are of particular importance because all of life occurs in such solutions, as well as a great deal of chemistry. Amidst this diversity, there seems only one uniformity: dielectric properties cannot be described by a single real dielectric constant.

In much higher frequency ranges, of light, for example, dielectric properties determine the refractive index, optical properties, and thus spectra of materials [37], because the polarization of electron orbitals determines how atoms absorb and radiate electromagnetic energy. Spectra are so varied that they are used as fingerprints to identify molecules [37, 39-43]. Spectral properties are as diverse as molecules and obviously cannot be described by a single constant refractive index.

Many interactions of light and materials cannot be described at all by dielectric constants. Dielectric constants are useful only when field strengths are small enough so polarization is a linear phenomenon, independent of field strength. Some of the most interesting applications of electrodynamics involve nonlinear, field dependent

¹ We use 'polarization' to mean any charge that depends on the local electric field, for example the charge in an highly polarizable anion like bromide Br^- or the nonuniform charge distribution of H_2O in the liquid state, with its complex time dependent (and perhaps nonlinear) polarization response to the local electric field. The charge or charge densities of alkali metal ions like Na^+ and K^+ are independent of the local electric field and so are not examples of polarization, but rather can be called permanent or fixed charge.

polarization [44-50].

The dielectric dilemma is clear: nonlinearities, spectra, and diverse dielectric behavior cannot be described by a single dielectric constant, but Maxwell's equations use a single dielectric constant, as they are usually written.

When a dielectric is complex, polarization and dielectric behavior need to be described by a functional, and the very form of the Maxwell equations changes. The detailed properties of polarization need to be known and described under the range of conditions in which polarization has significant effects. Polarization is rarely known that well experimentally. Theoretical models or simulations of that scope are also scarce.

Maxwell's equations with a single dielectric constant remain of great importance, however unrealistic the approximation, because that is how they have been taught ([1-5, 51, 52]; [50] is the exception) ever since the equations were formulated [53-56].

Students often remain unaware of the complex properties of the polarization of matter until they become scientists trying to use electrodynamics in applications. As scientists, they face a dielectric dilemma. Too little is often known of polarization to make the Maxwell equations useful in applications demanding precise understanding.

Conservation of current. We show here, however, that one property of electrodynamics is independent of polarization, namely conservation of current. That property is in fact enough to ensure that current flow in the one dimensional systems of our electronic technology can be described accurately by the simple, nearly algebraic equations of Kirchoff's current law, without dealing explicitly with field equations and their complexities, including singularities. The accuracy and simplicity of Kirchoff's current law help designers construct robust circuits that switch reliably in 10^{-9} sec. Singularities of field equations in three dimensions can increase sensitivity dramatically, making circuits less robust or reliable.

When current \mathbf{J}_{total} is defined to include the 'polarization of the vacuum' $\epsilon_0 \partial \mathbf{E} / \partial t$, conservation of current is entirely independent of the polarization and dielectric properties of matter. Lack of experimental knowledge of polarization is unimportant, as far as conservation of current \mathbf{J}_{total} is concerned.

The dielectric dilemma of the Maxwell equations is then partially resolved. Maxwell's equations can be used to analyze current flow in vacuum and matter, without knowledge of polarization. \mathbf{J}_{total} is conserved exactly, in vacuum and dielectrics, indeed in matter in general.

The resolution of the dielectric dilemma is only partial because electrical forces are not determined by currents alone. Forces depend on polarization even if current does not. Where forces are important, Maxwell's equations are useful only if polarization is

known in detail. Applications that depend on Kirchoff's current law can take advantage of conservation of current, without knowing anything about polarization or forces at all, as previously mentioned.

Proof. Here is the proof of these assertions, following [10, 57-62]. We define current as did Lorrain and Corson [1], p. 276 eq. 6-148.

$$\mathbf{J}_{total} = \mathbf{J} + \overbrace{\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}}^{\text{Vacuum Displacement Current}} \quad (1)$$

\mathbf{E} describes the electric field, with electric constant (permittivity of vacuum) ϵ_0 . \mathbf{J} describes all the movement of charge that has mass.

\mathbf{J} includes conduction currents (carried by electrons, holes, or ions, for example). \mathbf{J} includes all material polarization, no matter how transient and small are the underlying atomic motions. Thus, \mathbf{J} includes the dielectric current in its classical approximation $(\epsilon_r - 1)\epsilon_0 \partial \mathbf{E} / \partial t$ as well as all more complex material polarization currents. Here, ϵ_r is the relative dielectric coefficient of perfect dielectrics and classical electrodynamics, a single real positive constant ≥ 1 .

Polarization. Polarization of two types is important in our derivation. Polarization of matter ('material polarization') arises from the motion of charge, typically from the tiny, transient, reversible motion of electrons in insulators; or the movement of holes and electrons in semiconductors; or the distortion of electron orbitals around atoms or molecules in ionic solutions; or the rearrangement of molecules with asymmetrical distributions of permanent and induced charge, and some organized structure, like water.

Polarization of the vacuum $\epsilon_0 \partial \mathbf{E} / \partial t$ is qualitatively different from material polarization and is not a property of matter at all—although it exists in matter—since a vacuum contains no matter. It is not surprising that material models of current in a vacuum $\epsilon_0 \partial \mathbf{E} / \partial t$ are difficult to construct [53-56].

Polarization of the vacuum produces currents \mathbf{J}_{total} that allow electric and magnetic fields to propagate between stars at the velocity of light $c = 1/\sqrt{\epsilon_0 \mu_0}$, determined by the electrical and magnetic constants ϵ_0 , μ_0 and nothing else [1-5, 51, 52].

Polarization of the vacuum is a general property of 'the universe'. It arises from the properties of space-time as described by the theory of relativity [1-5, 51, 52]. The term $\epsilon_0 \partial \mathbf{E} / \partial t$, arises from a special property of charge. Charge is independent of

velocity, even at velocities approaching the speed of light, because the Lorentz transformation of charge creates the term $\epsilon_0 \partial \mathbf{E} / \partial t$ (as described in [63]; [2], p. 553; [1], p. 228, eq. 5-110). Mass, length and time all depend on velocity. It does not [2, 63]. Charge is special.

Maxwell's equations include Ampere's Law as Maxwell formulated it

$$\frac{1}{\mu_0} \mathbf{curl} \mathbf{B} = \mathbf{J}_{total} = \mathbf{J} + \overbrace{\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}}^{\text{Vacuum Displacement Current}} \quad (2)$$

$$\mathbf{J} = \underbrace{(\epsilon_r - 1)\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}}_{\text{Material Displacement Current}} + \mathbf{J}_{everything\ else} \quad (3)$$

\mathbf{B} describes the magnetic field. The polarization of idealized dielectrics $(\epsilon_r - 1)\epsilon_0 \partial \mathbf{E} / \partial t$ is isolated from other currents in eq. (2) only for convenience in dealing with the literature. $\mathbf{J}_{everything\ else}$ includes conduction currents (carried by electrons, holes, or ions, for example). $\mathbf{J}_{everything\ else}$ also includes all material polarization, no matter how transient and small are the underlying atomic motions. Thus, $\mathbf{J}_{everything\ else}$ includes the dielectric current in its classical approximation $(\epsilon_r - 1)\epsilon_0 \partial \mathbf{E} / \partial t$ as well as all the other properties of material polarization and classical conduction.

Conservation of Current. The divergence of the curl is zero whenever Maxwell's equations can be used, so conservation of current is as general as Maxwell's equations themselves.

$$\text{Conservation of Current} \quad \mathbf{div} \left(\overbrace{\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}}^{\text{Current}} \right) = 0 \quad (4)$$

This equation can be solved for \mathbf{E} . The solution shows that the electric field assumes the value needed to conserve current .

$$\mathbf{E}(x, y, z|t) = -\frac{1}{\epsilon_0} \int_0^t \mathbf{J}(x, y, z|\tau) d\tau \quad (5)$$

Electric field is an output. $\mathbf{E}(x, y, z|t)$ is not assumed. It is an output of the analysis. $\mathbf{E}(x, y, z|t)$ is consistent because it is the result of the integration of the Maxwell equation (2) and so depends on ‘everything’ in a system.

$\mathbf{E}(x, y, z|t)$ produces the polarization of the vacuum $\epsilon_0 \partial\mathbf{E}/\partial t$ needed to conserve total current \mathbf{J}_{total} , no matter what are the motions of matter that produce \mathbf{J} , no matter what are the properties of the material displacement current, including its classical approximation $(\epsilon_r - 1)\epsilon_0 \partial\mathbf{E}/\partial t$.

$\mathbf{E}(x, y, z|t)$ moves charges. The charge movement is a current that combines with $\epsilon_0 \partial\mathbf{E}/\partial t$ so Maxwell’s equations are satisfied exactly. $\mathbf{E}(x, y, z|t)$ describes the field that produces these charge movements. $\mathbf{E}(x, y, z|t)$ produces the forces and the material currents needed to satisfy Maxwell’s equations exactly, when they are combined with the material and vacuum displacement currents.

These words describe the mathematical fact that the Maxwell equations form a consistent theory, in which all variables—including $\mathbf{E}(x, y, z|t)$ and every form of current and flux—satisfy the equations and boundary conditions with one set of unchanging parameters.

In networks of circuit components, $\mathbf{E}(x, y, z|t)$ varies dramatically from component to component. The variation is not determined just by the local microphysics of conduction and material polarization but also by the global physics and structure of the network. $\mathbf{E}(x, y, z|t)$ depends on local microphysics, Maxwell equations, and boundary conditions that specify the global properties of the electric field.

The electric field is both global and local. This reality is most vivid in one dimensional networks where components are in series. Currents \mathbf{J}_{total} are equal in every component of a series system, at all times and in all conditions. The current in one component depends on the current in another. The microphysics of conduction in one component does not in itself determine the current flow through that component, despite our local intuition which might suggest otherwise.

Reference [58] discusses this property of series systems in detail, showing how the physics of each component consists of both the local microphysics specific to that component, and also the polarization of the vacuum, the displacement current $\epsilon_0 \partial\mathbf{E}/\partial t$. Fig. 3 of [58], and its discussion, show how $\mathbf{E}(x, y, z|t)$ varies in wires, resistors, capacitors, diodes, ionic solutions. $\mathbf{E}(x, y, z|t)$ varies in every component but \mathbf{J}_{total} is always the same, because the components are in series. The currents are equal in the components of the series circuit of Fig. 3 [58], at all times and in all conditions,

because the Maxwell equations produce the $\mathbf{E}(x, y, z|t)$ field, the material currents and fluxes, and the ‘vacuum’ displacement current $\epsilon_0 \partial\mathbf{E}/\partial t$ needed to conserve current \mathbf{J}_{total} , no matter what are the local microphysics of conduction or polarization [10, 58-60], no matter what the dielectric current is in its classical approximation $(\epsilon_r - 1)\epsilon_0 \partial\mathbf{E}/\partial t$.

Classical Treatments of Conservation of Current. The usual derivations [1-5, 51, 52], of conservation of current follow Maxwell [53-56] and use the composite field $\mathbf{D} = \epsilon_r \epsilon_0 \mathbf{E}$. The \mathbf{D} field is a composite because it includes a constitutive property describing the dielectric properties of matter. The \mathbf{D} field does not have a universal description independent of the properties of matter, because no universal description of the dielectric properties of matter seems possible.

Indeed, the \mathbf{D} field includes a severe approximation. It treats the dielectric constant ϵ_r as a single real constant number, in contradiction to experimental measurements of dielectric properties [6-36], spectra [39-43, 64], and nonlinear polarization [44-50]. When polarization has complex properties, the Maxwell equations as usually written are inadequate. They must be reformulated to describe the polarization that actually exists in the system of interest because that polarization significantly changes the outputs of the equations. Reformulation using the \mathbf{D} field becomes awkward, if not impossible in many cases, e.g., when polarization is time dependent or nonlinear. In those cases the form of the Maxwell equations changes and, from a mathematical point of view, they become a quite different set of partial differential equations, or integro-differential equations.

General statements of conservation of current, like eq. (4), are not easily found in the literature, probably because Maxwell formulated his equations in terms of the \mathbf{D} field, and others have followed in his footsteps, understandably enough.

The usual derivations of conservation of current describe dielectric properties inaccurately, so conservation of current appears to be inaccurate. Conservation of current appears to be a poor approximation that does not fit experimental data.

The appearance that conservation of current is an approximation is unfortunate because scientists are then reluctant to use the general statement of eq. (4) to understand electrical properties of matter with complex dielectric properties or polarization. Eq. (4) is particularly useful in understanding the physics of current flow in series arrangements like (semiconductor) diodes or ionic channels of biological membranes, where currents are the same everywhere, although the microphysics of conduction of charges is not the same at all.

Conservation of current is not an approximation, as we have just shown. Conservation of current is in fact a general and exact property of the Maxwell equations [10, 60, 62, 65], independent of any properties of matter, true under all conditions. Use of the general principle of conservation of current should allow scientists to better understand and control the flow of current in the circuit boards of our high speed digital devices, or in the crowded confines of protein channels in biological membranes, whether the channels conduct ions, electrons, or charged forms of water like H^+ or H_3O^+ .

Dielectric dilemma resolved. The dielectric dilemma posed by traditional formulations of Maxwell's equations is thus partially resolved. Current J_{total} is conserved. Applications that depend mostly on current conservation, like Kirchoff's current law for electrical networks, can use Maxwell's equations even when dielectric properties are unknown.

The dielectric dilemma is only partially resolved because forces depend on polarization even if current does not. Where forces are important, Maxwell's equations are useful in detail only if polarization is known in detail.

Engineering Applications. Engineering applications work reliably in dielectric environments that are rarely known in detail, over a remarkable range of times, starting around 10^{-10} sec in the digital circuits of our phones and computers.

Engineering applications use one dimensional systems in which current can be easily measured by (for example) measuring the potential drop across a one ohm (or smaller) resistor placed in series in a branch. All the components in series with that one ohm resistor will have the same current under all conditions and at all times [10, 57-60], independent of the dielectric complexities or conduction mechanism of current in the component. Currents in networks of branches are more complex but networks conserve current as it flows. Conservation of current takes the simple form of the algebraic equations called Kirchoff's current law, if current is defined as J_{total} [57]. Kirchoff's current law then allows exact analysis of one dimensional systems from the shortest times, because the current law is accurate whenever Maxwell's equations are valid.

It seems hardly a coincidence that so much of our technology uses one dimensional circuits where current can be accurately defined, measured, and controlled, even in nanoseconds. Significant efforts are made to keep the flow of current one dimensional in our high speed technology, particularly in its grounding systems [66], p.787 of [67].

One can hope that current flow in the channels of biology can some day be controlled and exploited (almost) as well as in our semiconductor technology today.

References

- [1] Lorrain P and Corson D 1970 *Electromagnetic fields and waves, Second Edition*: (Freeman)
- [2] Jackson J D 1999 *Classical Electrodynamics, Third Edition* (New York: Wiley)
- [3] Griffiths D J 1981 *Introduction to Electrodynamics* (Englewood Cliffs, NJ: Prentice Hall)
- [4] Purcell E M and Morin D J 2013 *Electricity and magnetism*: Cambridge University Press)
- [5] Feynman R P, Leighton R B and Sands M 1963 *The Feynman: Lectures on Physics, Mainly Electromagnetism and Matter* vol 2 (New York: Addison-Wesley Publishing Co., also at http://www.feynmanlectures.caltech.edu/II_toc.html)
- [6] Debye P J W 1929 *Polar molecules*: Chemical Catalog Company, Incorporated)
- [7] Debye P and Falkenhagen H 1928 *Dispersion of the Conductivity and Dielectric Constants of Strong Electrolytes*, *Phys* vol 29: Z)
- [8] Barsoukov E and Macdonald J R 2018 *Impedance spectroscopy: theory, experiment, and applications*: John Wiley & Sons)
- [9] Kremer F and Schönhals A 2003 *Broadband Dielectric Spectroscopy*: Springer)
- [10] Eisenberg B, Oriols X and Ferry D 2017 Dynamics of Current, Charge, and Mass *Molecular Based Mathematical Biology* **5** 78-115 and arXiv preprint <https://arxiv.org/abs/1708.07400>
- [11] Buchner R and Barthel J 2001 Dielectric Relaxation in Solutions *Annual Reports on the Progress of Chemistry, Section C: Physical Chemistry* **97** 349-82
- [12] Barthel J, Buchner R and Münsterer M 1995 *Electrolyte Data Collection Vol. 12, Part 2: Dielectric Properties of Water and Aqueous Electrolyte Solutions* (Frankfurt am Main: DECHEMA)
- [13] Kraus C A and Fuoss R M 1933 Properties of Electrolytic Solutions. I. Conductance as Influenced by the Dielectric Constant of the Solvent Medium1 *Journal of the American Chemical Society* **55** 21-36
- [14] Oncley J 1942 The Investigation of Proteins by Dielectric Measurements *Chemical Reviews* **30** 433-50
- [15] Fuoss R M 1949 Theory of dielectrics *Journal of Chemical Education* **26** 683
- [16] Von Hippel A R 1954 *Dielectric materials and applications* vol 2: Artech House on

Demand)

- [17] Fröhlich H 1958 *Theory of dielectrics: dielectric constant and dielectric loss*: Clarendon Press)
- [18] Nee T-w and Zwanzig R 1970 Theory of Dielectric Relaxation in Polar Liquids *Journal of Chemical Physics* **52** 6353-63
- [19] Scaife B K P 1989 *Principles of dielectrics*: New York, NY (USA); Oxford University Press; None)
- [20] Ritschel U, Wilets L, Rehr J J and Grabiak M 1992 Non-local dielectric functions in classical electrostatics and QCD models *Journal of Physics G: Nuclear and Particle Physics* **18** 1889
- [21] Kurnikova M G, Waldeck D H and Coalson R D 1996 A molecular dynamics study of the dielectric friction *Journal of Chemical Physics* **105** 628-38
- [22] Heinz T N, van Gunsteren W F and Hunenberger P H 2001 Comparison of four methods to compute the dielectric permittivity of liquids from molecular dynamics simulations *The Journal of chemical physics* **115** 1125-36
- [23] Pitera J W, Falta M and van Gunsteren W F 2001 Dielectric properties of proteins from simulation: the effects of solvent, ligands, pH, and temperature *Biophys J* **80** 2546-55
- [24] Schutz C N and Warshel A 2001 What are the dielectric "constants" of proteins and how to validate electrostatic models? *Proteins* **44** 400-17
- [25] Fiedziuszko S J, Hunter I C, Itoh T, Kobayashi Y, Nishikawa T, Stitzer S N and Wakino K 2002 Dielectric materials, devices, and circuits *IEEE Transactions on Microwave Theory and Techniques* **50** 706-20
- [26] Doerr T P and Yu Y-K 2004 Electrostatics in the presence of dielectrics: The benefits of treating the induced surface charge density directly *American Journal of Physics* **72** 190-6
- [27] Rotenberg B, Cadene A, Dufreche J F, Durand-Vidal S, Badot J C and Turq P 2005 An analytical model for probing ion dynamics in clays with broadband dielectric spectroscopy *The journal of physical chemistry. B* **109** 15548-57
- [28] Kuehn S, Marohn J A and Loring R F 2006 Noncontact dielectric friction *The journal of physical chemistry. B* **110** 14525-8
- [29] Dyer K M, Perkyuns J S, Stell G and Pettitt B M 2008 A molecular site-site integral equation that yields the dielectric constant *The Journal of chemical physics* **129** 104512

- [30] Fulton R L 2009 The nonlinear dielectric behavior of water: Comparisons of various approaches to the nonlinear dielectric increment *The Journal of chemical physics* **130** 204503-10
- [31] Angulo-Sherman A and Mercado-Uribe H 2011 Dielectric spectroscopy of water at low frequencies: The existence of an isopermittive point *Chemical Physics Letters* **503** 327-30
- [32] Ben-Yaakov D, Andelman D and Podgornik R 2011 Dielectric decrement as a source of ion-specific effects *The Journal of chemical physics* **134** 074705
- [33] Riniker S, Kunz A-P E and van Gunsteren W F 2011 On the Calculation of the Dielectric Permittivity and Relaxation of Molecular Models in the Liquid Phase *Journal of Chemical Theory and Computation* **7** 1469-75
- [34] Zarubin G and Bier M 2015 Static dielectric properties of dense ionic fluids *The Journal of chemical physics* **142** 184502
- [35] Eisenberg B and Liu W 2017 Relative dielectric constants and selectivity ratios in open ionic channels *Molecular Based Mathematical Biology* **5** 125-37
- [36] Böttcher C J F, van Belle O C, Bordewijk P and Rip A 1978 *Theory of electric polarization* vol 2: Elsevier Science Ltd)
- [37] Parsegian V A 2006 *Van der Waals Forces: A Handbook for Biologists, Chemists, Engineers, and Physicists* (New York: Cambridge University Press)
- [38] Israelachvili J 1992 *Intermolecular and Surface Forces* (New York: Academic Press)
- [39] Banwell C N and McCash E M 1994 *Fundamentals of molecular spectroscopy* vol 851: McGraw-Hill New York)
- [40] Demchenko A P 2013 *Ultraviolet spectroscopy of proteins*: Springer Science & Business Media)
- [41] Rao K N 2012 *Molecular spectroscopy: modern research*: Elsevier)
- [42] Sindhu P 2006 *Fundamentals of Molecular Spectroscopy*: New Age International)
- [43] Stuart B 2005 *Infrared spectroscopy*: Wiley Online Library)
- [44] Wegener M 2005 *Extreme nonlinear optics: an introduction*: Springer Science & Business Media)
- [45] Sutherland R L 2003 *Handbook of nonlinear optics*: CRC press)
- [46] Boyd R W 2008 *Nonlinear Optics, Third Edition*: Academic Press)
- [47] Hill W T and Lee C H 2008 *Light-matter interaction*: John Wiley & Sons)

- [48] Lodahl P, Mahmoodian S, Stobbe S, Rauschenbeutel A, Schneeweiss P, Volz J, Pichler H and Zoller P 2017 Chiral quantum optics *Nature* **541** 473-80
- [49] Zheng B, Madni H A, Hao R, Zhang X, Liu X, Li E and Chen H 2016 Concealing arbitrary objects remotely with multi-folded transformation optics *Light Sci Appl.* **5** e16177
- [50] Robinson F N H 1973 *Macroscopic electromagnetism* vol 57: Pergamon)
- [51] Abraham M and Becker R 1932 *The Classical Theory of Electricity and Magnetism* (Glasgow, UK: Blackie and subsequent Dover reprints)
- [52] Abraham M and Föppl A 1905 *Theorie der Elektrizität: Bd. Elektromagnetische Theorie der Strahlung* vol 2: BG Teubner)
- [53] Buchwald J Z 1985 *From Maxwell to Microphysics. Aspects of Electromagnetic Theory in the Last Quarter of the Nineteenth Century* (Chicago IL USA: University of Chicago)
- [54] Simpson T K 1998 *Maxwell on the Electromagnetic Field: A Guided Study*: Rutgers University Press)
- [55] Arthur J W 2008 The fundamentals of electromagnetic theory revisited *IEEE Antennas and Propagation Magazine* **50** 19-65
- [56] Arthur J W 2013 The Evolution of Maxwell's Equations from 1862 to the Present Day *IEEE Antennas and Propagation Magazine* **55** 61-81
- [57] Eisenberg B, Gold N, Song Z and Huang H 2018 What Current Flows Through a Resistor? *arXiv preprint arXiv:1805.04814*
- [58] Eisenberg R S 2016 Mass Action and Conservation of Current *Hungarian Journal of Industry and Chemistry* *Posted on arXiv.org with paper ID arXiv:1502.07251* **44** 1-28
- [59] Eisenberg B 2016 Conservation of Current and Conservation of Charge *Available on arXiv as <https://arxiv.org/abs/1609.09175>*
- [60] Eisenberg B 2016 Maxwell Matters *Available on arXiv as <https://arxiv.org/pdf/1607.06691>*
- [61] Benseny A, Albareda G, Sanz Á S, Mompert J and Oriols X 2014 Applied bohmian mechanics *The European Physical Journal D* **68** 286
- [62] Albareda G, Traversa F L, Benali A and Oriols X 2012 Computation of Quantum Electrical Currents through the Ramo-Shockley-Pellegrini Theorem with Trajectories *Fluctuation & Noise Letters* **11** 1-11

- [63] Pauli W 2013 *Theory of Relativity*: Dover Publications)
- [64] Parsegian V A 1969 *Nature (London)* **221** 844
- [65] Marian D, Colomes E and Oriols X 2015 Time-dependent exchange and tunneling: detection at the same place of two electrons emitted simultaneously from different sources *J Phys Condens Matter* **27** 245302
- [66] Joffe E B and Lock K-S 2010 *Grounds for Grounding* (NY: Wiley-IEEE Press)
- [67] Paul C R 2006 *Introduction to electromagnetic compatibility* vol 184: John Wiley & Sons)