Maxwell's Core Equations

Conservation of Total Current

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Workshop on
Mathematics in Action:
Modeling and Analysis in Molecular Biology and Electrophysiology

Abstract

Ionic solutions determine many properties of living systems, nanosystems, and electrochemistry, including batteries. Ionic solutions are customarily analyzed in the tradition of thermodynamics extended by statistical mechanics. The statistical mechanics used is based on the theory of a perfect gas, in which point particles, without internal structure, let alone internal motions or dissipation, interact only with elastic collisions, isotropic at that, in the thermodynamic limit, in which boundary conditions do not appear, let alone confine. None of these assumptions apply to ionic solutions, even approximately, because molecules and water have internal structure, dissipate energy in inelastic anisotropic collisions. Indeed, ionic solutions are usually confined by boundary conditions that form devices of the greatest practical importance, like nerve fibers, cardiac muscle, and batteries. Devices are defined by their input output relations that are specified by boundary conditions in finite domains. The boundary conditions involve flow and do not exist in the thermodynamic limit.

Ions and water in ionic solutions interact by electrodynamics, including time dependent terms important on the scale of atoms. Ions interact strongly on all scales, and their interactions are not confined. Potentials on boundaries very far away create flows that allow living systems and batteries to function, often as devices. Electrodynamic interactions extend to infinity thanks to $\varepsilon_0\,\partial\boldsymbol{E}/\partial t$, as a glance at the sun or stars demonstrates. The crucial biological phenomenon of the action potential arises from the coupling of macroscopic fields with atomic scale movements of ions and channel proteins. Classical statistical mechanics can deal with none of this without electrodynamics that extend far beyond the thermodynamic limit

Classical electrodynamics may seem at first glance as weak a foundation for theories and simulation as statistical mechanics. After all, classical electrodynamics includes a dielectric constant that is ill defined by reality, since no ionic solution has dielectric properties that can be reasonably approximated by a single positive constant. Once the dielectric constant is removed—or should I say exorcized?—mechanical models are needed to describe the response of charge to force. These models are of course specific and approximate, the opposite of universal and exact.

Electrodynamics without a dielectric constant is quite different from statistical mechanics. To my considerable surprise, it provides a firm foundation for theory and simulation. Easily measured properties of charge movement are described by a universal and exact law, for ***any*** motions of matter or charge. Conservation of total current—that includes $\varepsilon_0\,\partial\boldsymbol{E}/\partial t$—is as universal and exact as the Maxwell equations themselves.

Written in one dimension, conservation of total current can become equality of total current, so total current in ion channels or circuit components can be described by ordinary differential equations in time, without a spatial variable at all, even for nearly Brownian currents of thermal motion! Conservation of current becomes Kirchhoff's law for ***total*** current that allows analysis of the action potential and the the design of integrated circuits, digital and analog.

Scientists are trained to be skeptical of universals. Scientists are trained to expect the particular and its parameters. To their discomfort, scientists see that the Maxwell equations without a dielectric constant are both exact and universal, without material parameters. Material parameters are found in abundance in the models of charge movement needed to describe real ionic solutions.

It might be wise to build models of ionic solutions on the firm foundation of exact electrodynamics, ***abandoning the quicksand of statistical mechanics, and dielectric constants, using instead explicit models*** of motions of charge and mass coupled to the Maxwell equations that describe electrodynamics everywhere, from inside atoms to between stars.

昆山杜克大学
DUKE KUNSHAN UNIVERSITY

Mathematics in Action

Special Thanks

to

Shixin Xu

and all the organizers

and to Yiewei Xiong for her hard work!

Essence of Electrodynamics

is

Maxwell's Core Equations

for the

Flows and Forces of Charge and Current
in matter and space

1) Maxwell Equations for Material Systems. doi: 10.20944/preprints202011.0201.v1 10.20944/preprints202011.0201.v1;
2) Thermostatics vs. Electrodynamics. DOI: 10.20944/preprints202009.0349.v2
3) Maxwell Equations Without a Polarization Field, Using a Paradigm from Biophysics. Entropy 23 172,
also available on arXiv at 2009.07088.pdf and 07010.03390/e23020172

Maxwell's Core Equations

Describe Electricity with no errors, <10^{-6}

almost everywhere at any time

$$\mathbf{div\ E} = \frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}} \qquad \mathbf{div\ B} = \mathbf{0}$$

Relativistic Property of Space NOT matter

$$\mathbf{curl\ E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \mathbf{curl\ B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

E is electric field, **B** is magnetic field

J is the current of all mass, including brief dielectric transients of the **P** and **D** fields

ρ is charge density (of all types, including dielectric charge of the **P** and **D** fields)

ε_0 is the permittivity of a vacuum

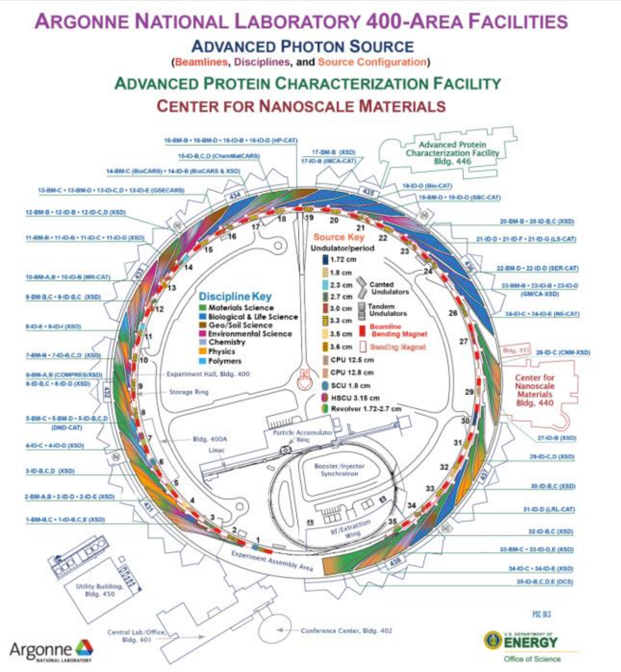
μ_0 is the permeability of a vacuum

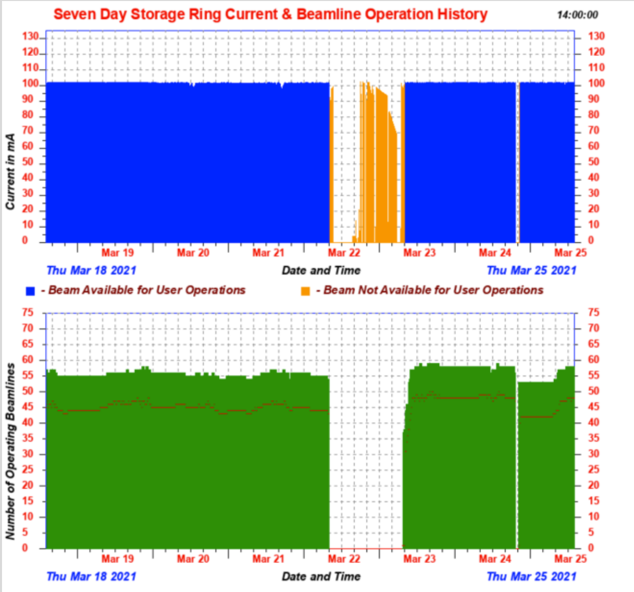
$(\mu_0 \varepsilon_0)^{-1/2}$ = velocity of light (!)

Advanced Photon Source Argonne National Laboratory

Error in Theory
$< 10^{-10}$

Beam $\sim 10^{10}$ eV
Beam length 10^{3} m
Tolerance $< 10^{-7}$ m
Beam Current 100 mA
Beam Power 10^{9} watts





Maxwell's Core Equations are Universal and Exact

but they are a Set of Coupled Partial Differential Equations that

need sophisticated mathematics and computations to describe systems of complex structure

Electro 'statics'

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

Magneto 'statics'

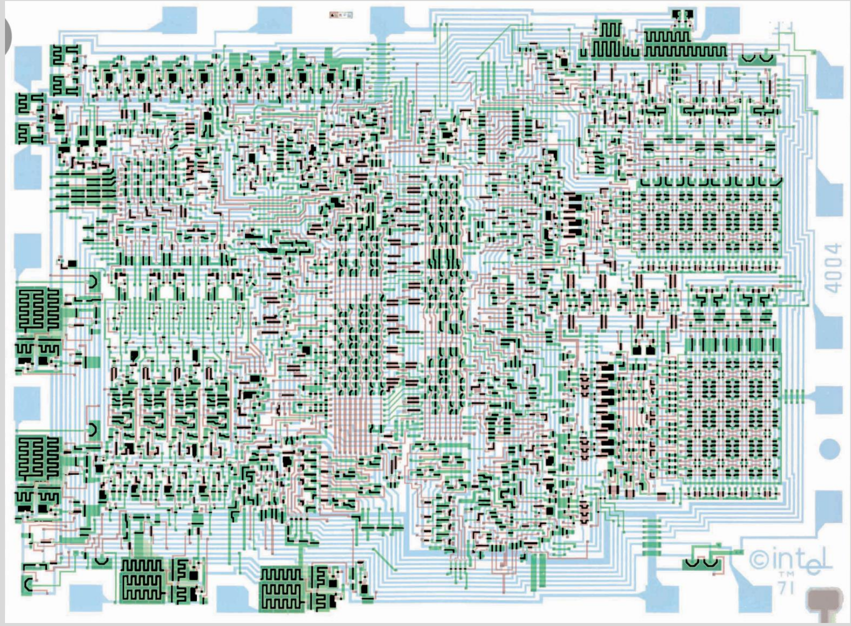
$$\nabla \cdot \mathbf{B} = 0$$

Electrodynamics

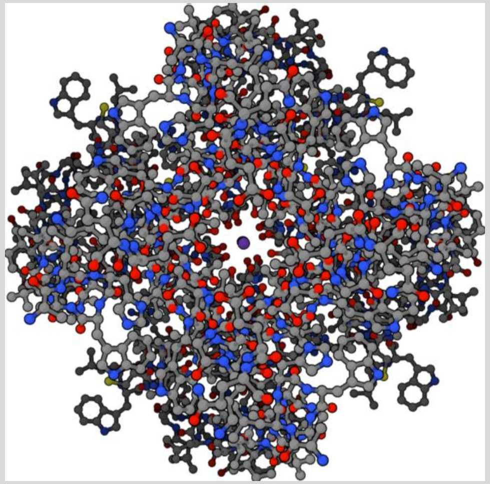
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Magnetodynamics

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$



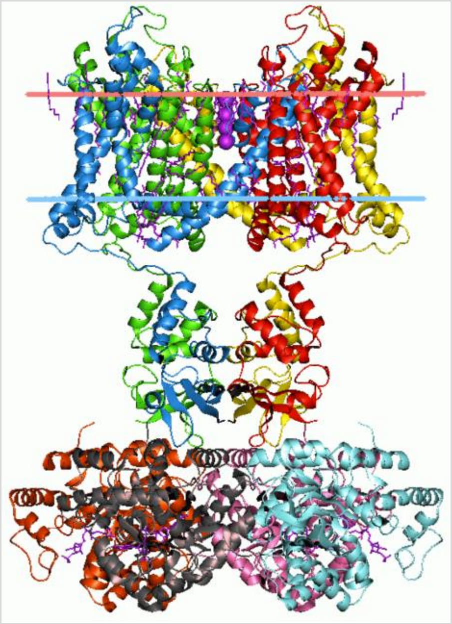
Integrated Circuit



Potassium Ion Channel $K_{V_{1.2}}$

PDB: **1BL8**

or



Scientists are Taught to be Skeptical

as they should be

particularly of Universal Exact Theories

like the Maxwell Core Equations

Richard Feynman

"Whenever you see a **sweeping statement** that a tremendous amount can come from a very **small number of assumptions**,

You always find that it is False.

There are usually a large number of

Implied Assumptions

that are far from obvious if you think about them sufficiently carefully."

Section 26-1.

The Feynman Lectures on Physics *Vol 2 (*1963) also at ***http://www.feynmanlectures.caltech.edu/II_toc.html***

Challenge to Audience

What are implied assumptions?

in Conservation of Total Current and Core Maxwell Equations

Contact Bob.Eisenberg@gmail.com

Don't Recognize these Maxwell Equations?
Where is the dielectric constant ε_r?

$$\mathrm{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \mathrm{div}\,\mathbf{B} = 0$$

Relativistic Property of Space NOT matter

$$\mathrm{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \mathrm{curl}\,\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

$\mathbf{J}$ is current of ALL mass, including tiny, brief dielectric transients of the $\mathbf{P}$ and $\mathbf{D}$ fields

ρ is density of ALL types of charge with mass including polarization and dielectric charge

Move the Physics of Dielectrics into J and ρ

$$\mathrm{div}\,\mathbf{E} = \frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}} \qquad \mathrm{div}\,\mathbf{D} = \mathrm{div}\,\boldsymbol{\varepsilon_r}\varepsilon_0\mathbf{E} = \rho_f$$

$$\frac{1}{\mu_0}\mathbf{curl}\,\mathbf{B} = \mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t} \qquad \frac{1}{\mu_0}\mathbf{curl}\,\mathbf{B} = \tilde{\mathbf{J}} + \boldsymbol{\varepsilon_r}\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

Polarization is part of J and ρ

Why Change Maxwell Equations?

Why Move the Physics of Dielectrics into $\mathbf{J}$ and ρ?

Dielectric Dilemma. arXiv: 1901.10805 (2019)

Dielectric Model does not fit data from Ionic Solutions

L.F. Lima et al. / Journal of Molecular Liquids 241 (2017) 530–539

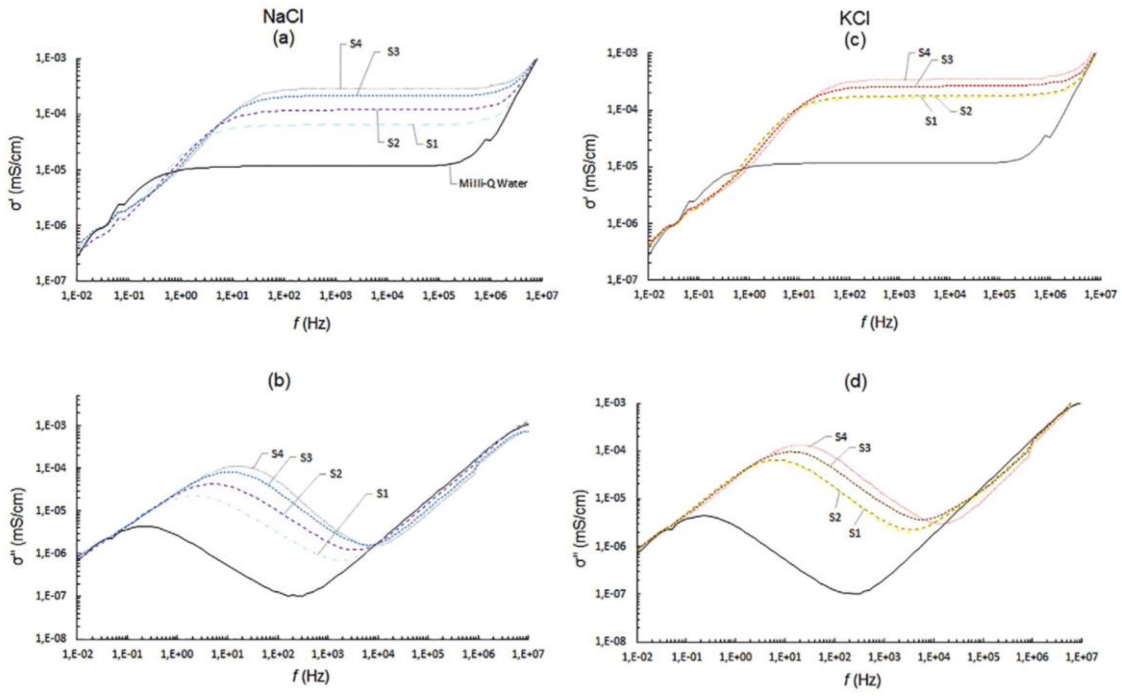


Fig. 4. Real and imaginary parts of the complex dielectric conductivity. (a) σ' and (b) σ'' of $NaCl_{(aq)}$; (c) σ' and (d) σ'' of $KCl_{(aq)}$.

Bulk (homogeneous, unstructured) Ionic Solution

Electrical Systems need Models of How Electrical Force Changes Charge Distribution

Dielectric Model does not fit data from Proteins

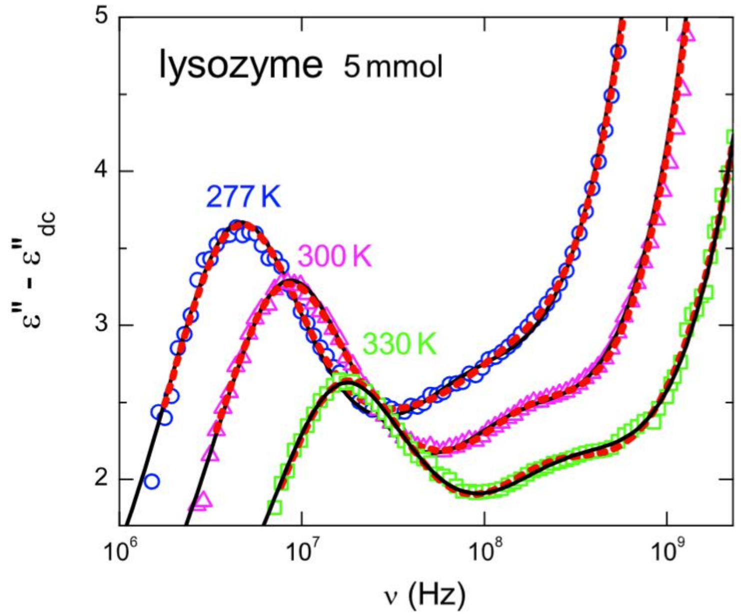
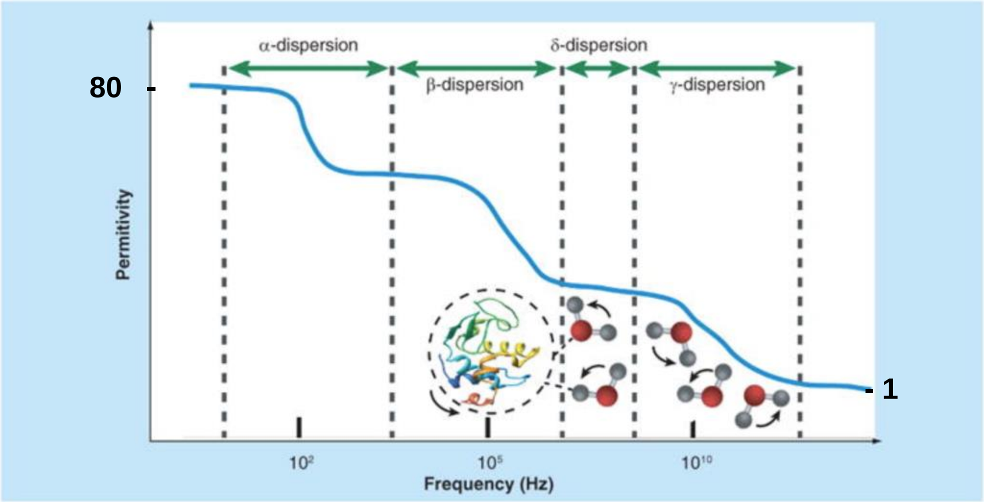


Fig. 3. Dielectric-loss spectra of a 5 mmol lysozyme solution in the region of the β- and δ-relaxations at different temperatures. The solid lines are fits using the sum of a Debye function for the β-relaxation and two Cole–Cole functions for the δ- and γ-relaxations. Dashed lines represent fits with four Debye functions according to Ref. [4].

**Wolf, Gulich, Lunkenheimer & Loidl. 2012.
Biochimica et Biophysica Acta (BBA) –
Proteins and Proteomics 1824:723-730.**



Complex permittivity spectrum of a typical protein solution showing distinct dispersions at their respective frequency range

**Electrical Systems need
Models of
How Electrical
Force Changes Charge Distribution**

Dielectric Model does <u>not</u> fit Data from Molecules

with Steric Excluded Interactions*

<u>which are nearly universal in proteins and ionic solutions</u>

Gudarzi and Aboutalebi (2021)
Self-consistent dielectric functions of materials:
Toward accurate computation of Casimir–van der Waals forces.
Science Advances 7:eabg2272.

also

Dzyaloshinskii, Lifshitz, and Pitaevskii. 1961. The general theory of van der Waals* forces. Advances in Physics 10:165-209.
Reyes-Coronado, Ortíz-Solano, Zabala, Rivacoba, and Esquivel-Sirvent. 2018. Analysis of electromagnetic forces and causality in electron microscopy. Ultramicroscopy 192:80-84.

Electrical Systems need Explicit Models of how electrical force changes charge distribution

as a function of time, location and conditions

The electric force pushes atoms with charge, and they move.

Electrodynamic Systems need

Explicit Models

of how electrical force changes charge distribution,

as function of time, position, conditions

not just a dielectric constant.

1) Maxwell Equations for Material Systems. doi: 10.20944/preprints202011.0201.v1 10.20944/preprints202011.0201.v1;
2) Thermostatics vs. Electrodynamics. DOI: 10.20944/preprints202009.0349.v2
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General Statement of Polarization of Charge is Impossible

Just as a

General Statement of Stress Strain Relations of Matter is Impossible

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<u>General</u> Statement of Polarization of Charge is Dangerous

Just as a
General Statement of Stress Strain Relations of Matter is Dangerous

because

<u>Specific</u> Properties of Polarization are the ESSENTIAL Physics

Just as a
Specific Properties of Stress Strain Relations are the ESSENTIAL Physics

Nothing to Adjust in Maxwell Core Equations

No implicit parameters or assumptions are visible

$$\mathbf{div\,E} = \frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}} \qquad \mathbf{div\,B} = \mathbf{0}$$

$$\mathbf{curl\,E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \mathbf{curl\,B} = \mu_0\left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

Scary !

Corollaries of Maxwell Core Equations Are also Scary: nothing to adjust

1) Conservation of Total Current

$$\mathbf{div}\left(\overbrace{\mathbf{J}+\ \varepsilon_0\,\partial\mathbf{E}/\partial t}^{\boldsymbol{Total\,Current}}\right)=0$$

2) Continuity equation

$$\mathbf{div}\,\mathbf{J}=\overset{\boldsymbol{All\,Charges}}{-\,\partial\rho/\partial t}$$

3) Wave equation

$$\mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}-\nabla^2\mathbf{E}=0$$

Speed of Light

Corollary of Maxwell Equations

Conservation of Total Current

Derivation

Maxwell: $\mathbf{curl\ \ B} = \mu_0(\mathbf{J} + \varepsilon_0\, \partial\mathbf{E}/\partial t)$

Theorem: $\mathbf{div\ curl\ \ B = 0}$

Imply the Corollary: $\mathbf{div}\, \underbrace{\mu_0(\mathbf{J} + \varepsilon_0\, \partial\mathbf{E}/\partial t\,)}_{\text{Total Current} = \mathbf{J}_{\mathbf{total}}} = 0$

Corollary

Total Current is conserved: $\boxed{\mathbf{div\ J_{total} = 0}}$

$\mathbf{J}$ = Flux of **all** Charges with mass, however small, fast, or transient

$\mathbf{J}$ is not conserved. It accumulates according to the continuity equation $\mathbf{div\, J} = -\,\partial\rho/\partial t;$ ρ is **all** charge

Conservation of Total Current

$$\mathbf{div}\,(\,\mathbf{J} + \varepsilon_0\,\partial\mathbf{E}/\partial t\,) = 0$$

Flux of all charge with mass

Relativistic Property of Space NOT matter

Scary: nothing to Adjust

Note: there are ZERO adjustable or vaguely defined parameters

$\mathbf{J}$, $\mathbf{E}$, $\mathbf{J}_{\mathbf{total}}$ are routinely measured in laboratories.

$\mathbf{J}_{\mathbf{total}}$, $\mathbf{E}$ control electronic circuits and biological cells

Why is Conservation of Total Current Universal and Exact

even in a vacuum?

Because $\varepsilon_0 \partial \mathrm{E}/\partial t$ flows in a vacuum

$\boxed{\varepsilon_0 \partial \mathrm{E}/\partial t \text{ is a Property of Space}}$

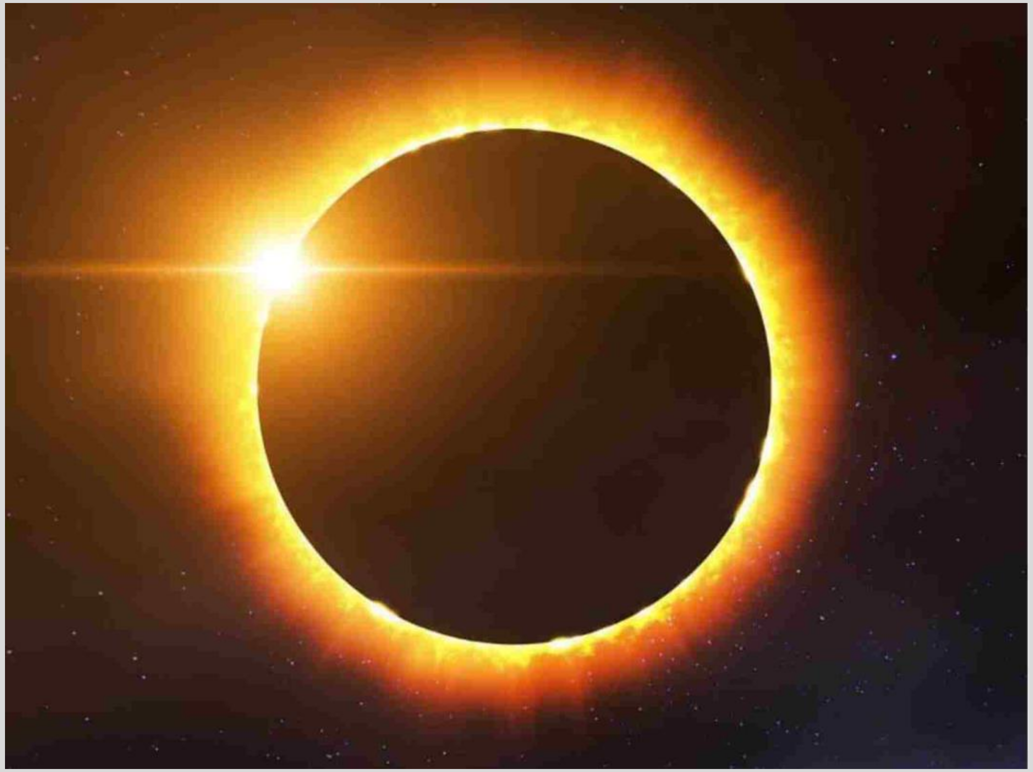
$\varepsilon_0 \partial \mathrm{E}/\partial t$ in a perfect vacuum produces B field

$\varepsilon_0 \partial \mathrm{E}/\partial t$ in a perfect vacuum produces light waves

$\varepsilon_0 \partial \mathrm{E}/\partial t$ is called the ‘ethereal current’ or the ‘displacement current’

because an ‘ether’ was once thought to fill everything, vacuum and empty space included

Jeans 1908. The mathematical theory of electricity and magnetism.
Whittaker 1951. A History of the Theories of Aether & Electricity.
Simpson 1998. Maxwell on the Electromagnetic Field: A Guided Study.



Ethereal current $\varepsilon_0 \partial \mathrm{E}/\partial t$ flows in vacuum of space,

once thought to be filled with an 'ether'

Jeans 1908. The mathematical theory of electricity and magnetism.
Whittaker 1951. A History of the Theories of Aether & Electricity.
Simpson 1998. Maxwell on the Electromagnetic Field: A Guided Study.

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

Wave Equation
Corollary of
Maxwell Equations

Velocity of light

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

Conservation of Total Current

Universal and Exact

Inside atoms, and between stars

because

It arises from the Lorentz invariance of charge

in the relativity theory of locally inertial systems

Proof is in every textbook and
Dunstan (2008) Phil Trans Roy Soc A 366: 1861

Charge does not vary when velocity approaches the speed of light

Length, time, (relativistic) **mass do vary**

Conservation of total current is not a property of mass.
It is a property of space.

Conservation of Total Current
is
EQUALITY of Total Current in a Channel or Component

Well known in Electronics

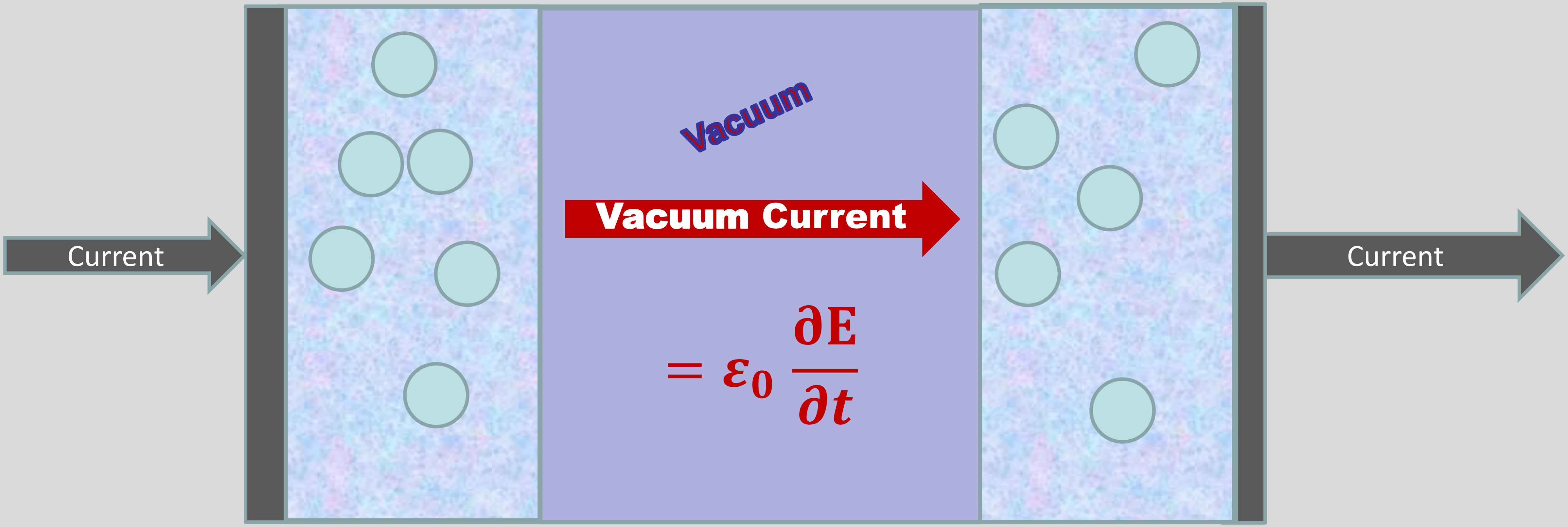
"It is, after all, the **sum** of electron current and **displacement** current which has **no divergence**. One of those two components can take over from the other."

Landauer (1992) Physica Scripta T42 p 110.

"**Electrodynamic** fields are endowed by **unique** features, including an **exquisite spatial nonlocality**"

Slight paraphrase of Lundeberg *et al* (2017)
Tuning quantum nonlocal effects plasmonics
Science 357:187-191

EQUALITY of Total Current is Well known in First Year Physics



Vacuum current = Ethereal current = Displacement Current
All are names for the same thing $\varepsilon_0\, \partial E/\partial t$

Godin and Botvinovskii. 2009. Measurement of displacement currents. Journal of Communications Technology and Electronics 54:1092-1095.

If Total Current is the same every place in a component or channel,

it does not depend on location.

Differential equation in x is not needed for $\mathrm{J} + \varepsilon_0\, \partial E/\partial t$

in channels or circuit components.

‘Current is the same at all x’
means
‘an equation in x is not needed’.

The two statements have the same logical content.

‘Total Current flow is perfectly smooth in the spatial coordinate’
is another statement of the same thing.

Equality of Total Current

can create a

Perfect Low Pass Spatial Filter.

It can convert

Infinite Spatial Variation **of Brownian Motion**

into a ***Constant in Space.***

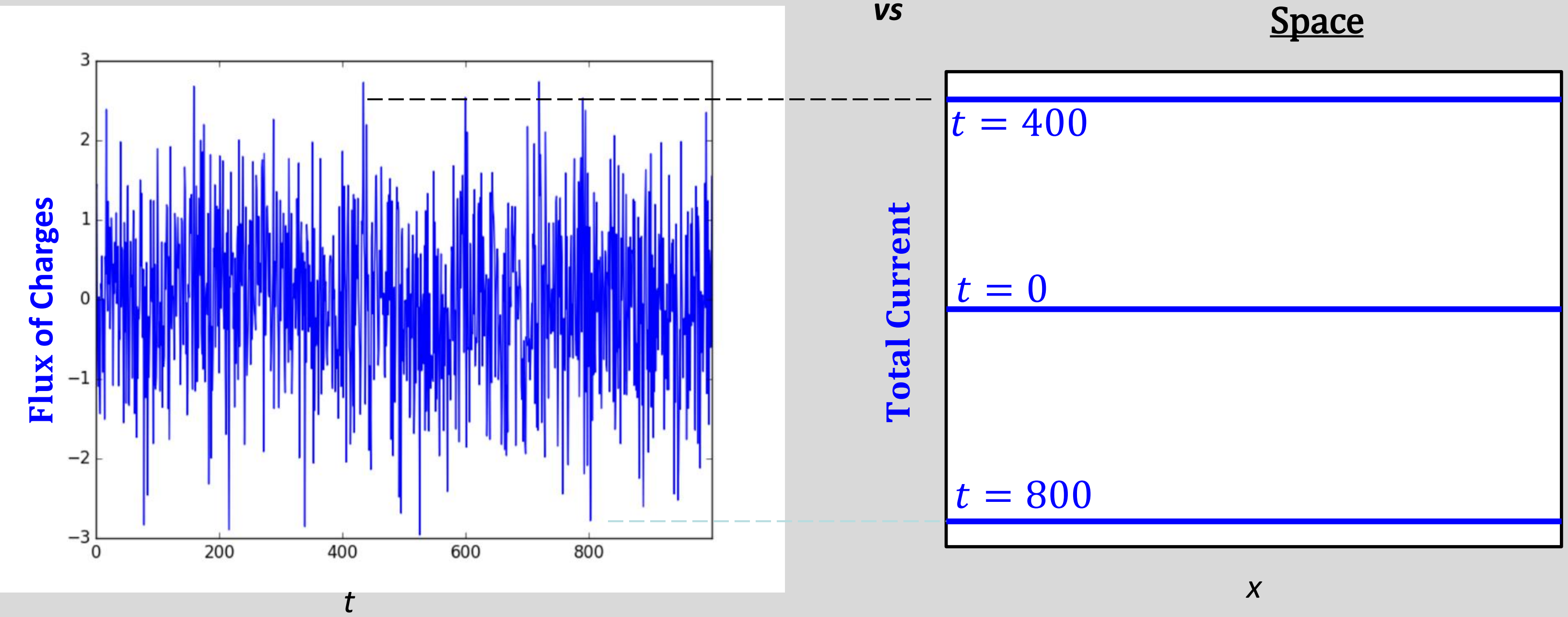
In that sense it is an

****Infinite Simplification***

Total Current Noise is Zero in Space

Infinite Simplification in Space

Time vs Space



One Dimensional Systems like Channels or Circuit Components

Not Widely Known
In Biophysics

Equality of Current in a Channel

implies

Total Current is independent of location in a channel or component

Total Current does NOT flow by hopping

Particles can hop, but total current cannot!

Entropy (2021) 23 172; doi (2020): 10.20944/preprints202011.0201.v1; arXiv (2021) 2002.09012

Well known in Electronics

"Hopping Models Ignore Capacitive Currents"

Paraphrase from Landauer (1992)

Conductance from transmission: common sense points.

Physica Scripta 1992 p.110

Capacitive Currents are Enormous

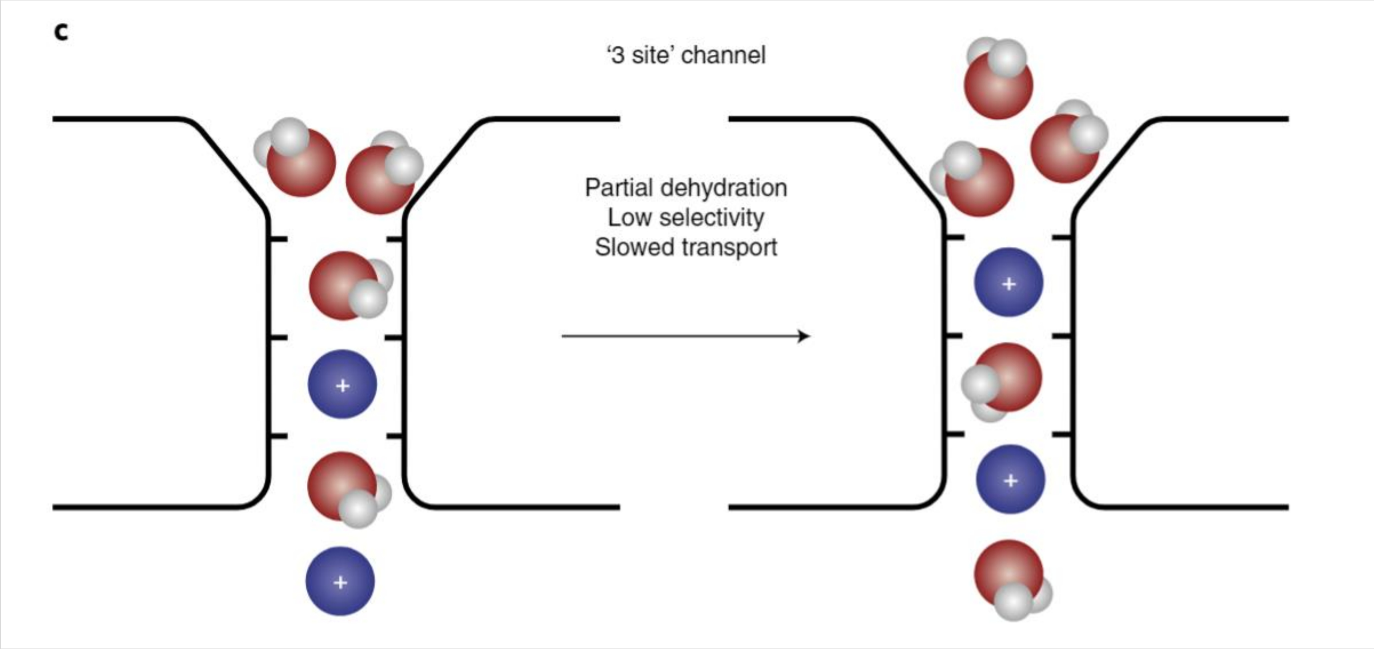
on the atomic scale of distance and time

$10^{-11}\text{m} \;\; 10^{-15}\text{s}$

Paradigm Change

Knock On and Knock Off of Ions does not describe behavior of Total Current Through the Channel because Total Current is the Same Everywhere!

arXiv 2002.09012 (2020); DOI: 10.20944/preprints202009.0349.v2



Corry (2018) The naked truth about K^+ selectivity. Nature Chemistry 10:799-800.

Not Widely Known

Particle motion itself does NOT define Current

<u>Contradicts Intuition</u>

Total Current must always be described by Continuum Equations

because it includes $\varepsilon_0 \partial \mathbf{E} / \partial t$

Total Current $\neq$ Flux of charge
Flux is defined only by particle motion

Another <u>General Implication</u> Of Conservation Of Total Current

Not Widely Known

Particle Theories

like Molecular Dynamics

REQUIRE

Continuum Description

of

Electric Current

because they must include ethereal current

$$\varepsilon_0 \frac{\partial \mathrm{E}}{\partial t}$$

Or they cannot satisfy Maxwell equations even approximately.
$\varepsilon_0 \partial \mathrm{E}/\partial t$ is enormous on atomic scale of molecular dynamics simulations
Description of Particle Motion is not enough to define Total Current.

Conservation of Total Current
is
an Enormous Simplification

Allows an exact Kirchhoff Current Law

Eisenberg, Gold, Song, and Huang. 2018. What Current Flows Through a Resistor? arXiv preprint arXiv:1805.04814.
Eisenberg, R. S. 2019. Kirchhoff's Law can be Exact. arXiv preprint available at https://arxiv.org/abs/1905.13574.

What does Conservation of Total Current mean for theory and simulations?

Current flow is very smooth in spatial coordinate
Differential equation in x is not needed for $\mathbf{J} + \varepsilon_0\, \partial E/\partial t$

What does this mean for theory and simulations?

YOU tell me!

Opportunity to Simplify Algorithms and Codes

perhaps dramatically

Spatial Dependence is Already Known
Only have to average the time dependence of particle motion
Ma, Li and Liu (2016). arXiv:1605.04886; Ma, Li and Liu (2016). arXiv:1606.03625.

What does Conservation of Total Current mean for practical engineers and biophysicists?

Note

Total Current is of Practical Importance

‘Total Current’ is the biological variable that drives the action potential of nerve and muscle.

‘Total current’ is the key physical variable manipulated in electronic circuits.

If Applications Depend only on Currents

Do not need to know location of charges!

Electric circuit theory and semiconductor device design depends on current laws, Kirchhoff's law, in particular. They do not involve location of charges!

Conservation of Total Current is then An Enormous Simplification.

Flux of all charge with mass

Relativistic Property of Space NOT matter

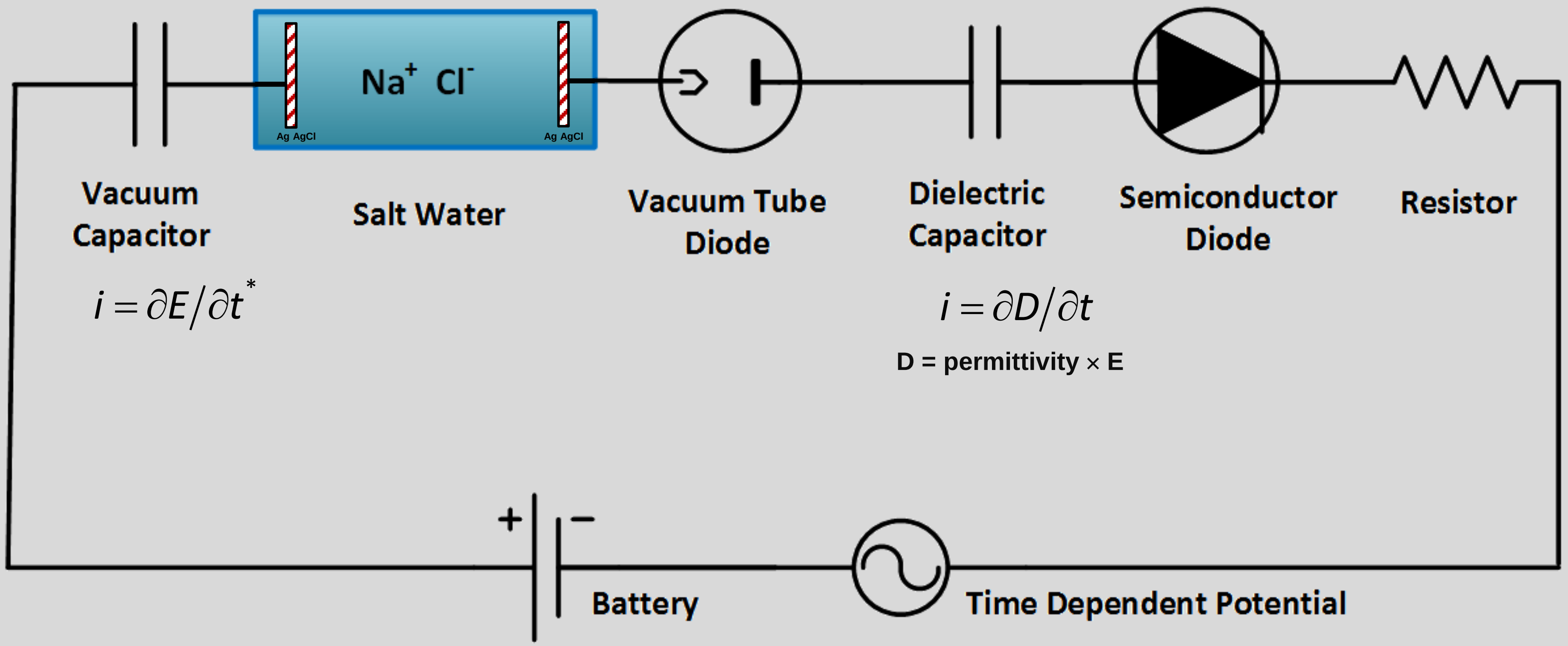
$$\mathbf{div}\left(\overbrace{\mathbf{J}+\ \varepsilon_0\,\partial\mathbf{E}/\partial t}^{Total\ Current}\right)=0$$

Another General Implication

Conservation of Total Current is Exact

even though
Physics of Charge Flow
Varies Profoundly

Flux and Total Current have very Different Physics
in Different Systems



but

Continuity of Total Current is Exact

$$J_{\text{Device 1}} = J_{\text{Device 2}} = J_{\text{Device 3}} \cdots$$

no matter what carries the current, at all times and all locations!

Mass Action and Conservation of Current. Hungarian Journal of Industry and Chemistry 44:1-28, also at arXiv:1502.07251

Another General Implication

Conservation of Total Current is Exact

Even though
Physics of Charge Flow
Varies Profoundly

How can that possibly be?

Total Current is conserved because

Electrodynamic Fields $\mathbf{E}$, $\mathbf{B}$ take on the Values that Conserve Current because they are solutions of Maxwell Equations

so total current $\mathbf{J}(x,t) + \varepsilon_0\, \partial \boldsymbol{E}/\partial t$ is always conserved

as required by a Corollary of the Maxwell equations

Details and PROOF
including quantum mechanics at
Eisenberg, Oriols, and Ferry. 2017. Dynamics of Current, Charge, and Mass.
Molecular Based Mathematical Biology 5:78-115
and arXiv https://arxiv.org/abs/1708.07400

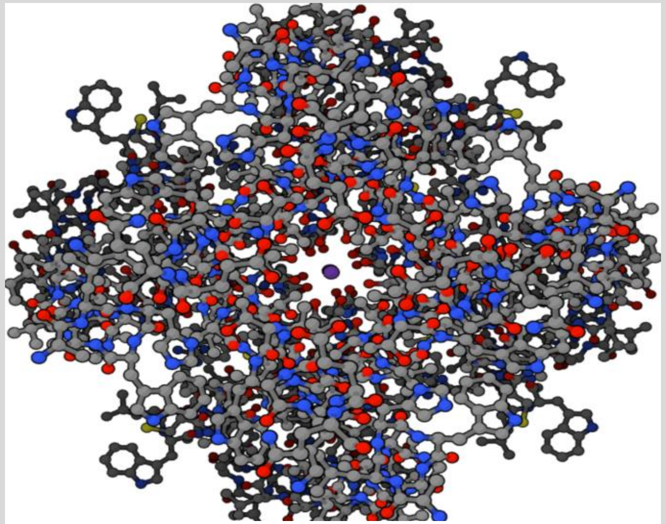
This is NOT mysterious

E is a force field that moves atoms
So Maxwell Equations are Satisfied

Details and PROOF
including quantum mechanics at
Eisenberg, Oriols, and Ferry. 2017. Dynamics of Current, Charge, and Mass.
Molecular Based Mathematical Biology 5:78-115
and arXiv https://arxiv.org/abs/1708.07400

E is a force field that moves atoms

in proteins



Potassium Ion Channel $K_{V_{1.2}}$
PDB: **1BL8**

so total current $\mathbf{J}(x,t) + \varepsilon_0\, \partial \boldsymbol{E}/\partial t$ is always conserved
so the Maxwell equations are always satisfied.

Details and PROOF
including quantum mechanics at
Eisenberg, Oriols, and Ferry. 2017. Dynamics of Current, Charge, and Mass.
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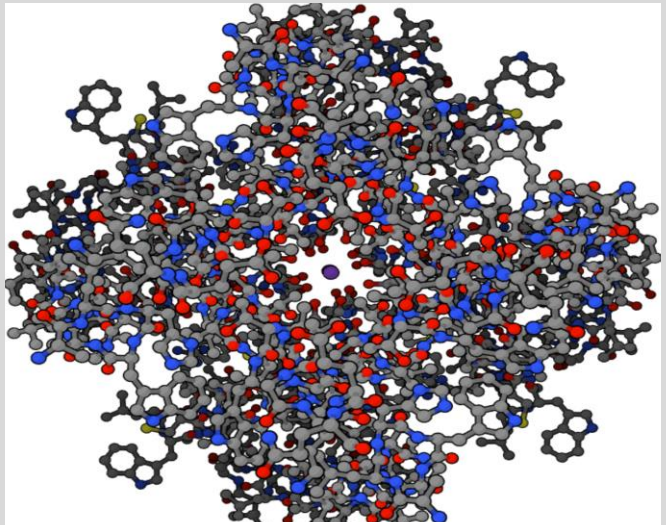
Most Movement of Atoms Is Produced By E Field That Conserves Total Current

in proteins



Potassium Ion Channel $K_{V_{1.2}}$
***PDB:* 1BL8**

Details and PROOF
including quantum mechanics at
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Molecular Based Mathematical Biology 5:78-115
and arXiv https://arxiv.org/abs/1708.07400

If location of charges is needed, another corollary of the Maxwell Equations is involved,

Continuity Equation

$$\mathbf{div}\,\mathbf{J} = -\frac{\partial}{\partial t}\rho\left(x, y, z \middle| t\right)$$

ρ describes all charges

$\mathbf{J}$ is the flux of ρ

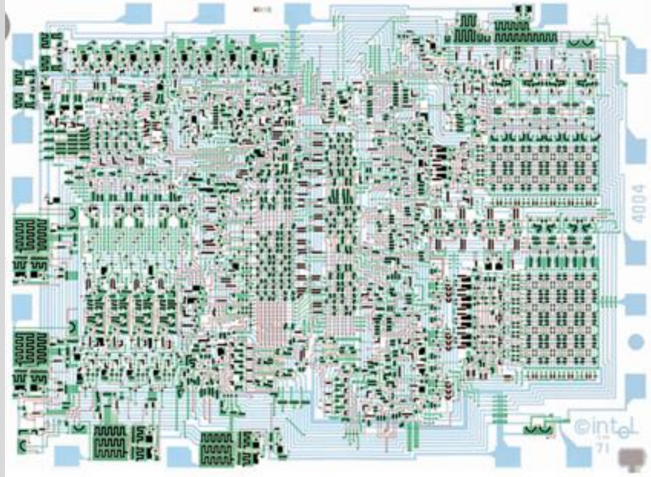
Charges are Described by Another Corollary of Maxwell

Continuity Equation is not very useful

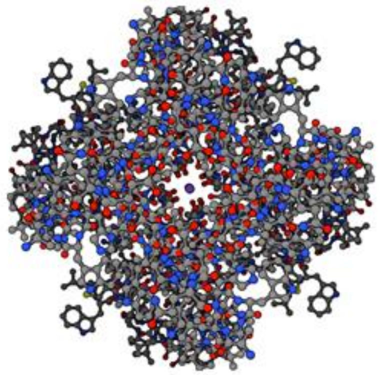
It depends on Feynman's 'Hidden Implications'

Must know all charges and how they move

$$\mathbf{div\ J} = -\frac{\partial \rho(x, y, z|t)}{\partial t}$$

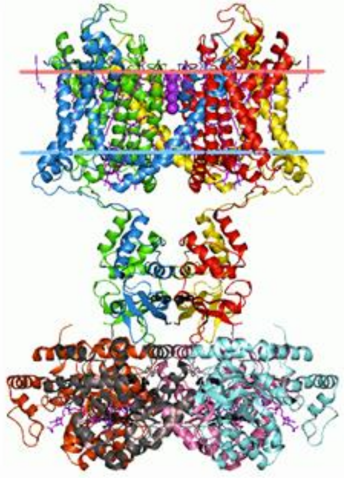


Integrated Circuit



Potassium Ion Channel $K_{V_{1.2}}$

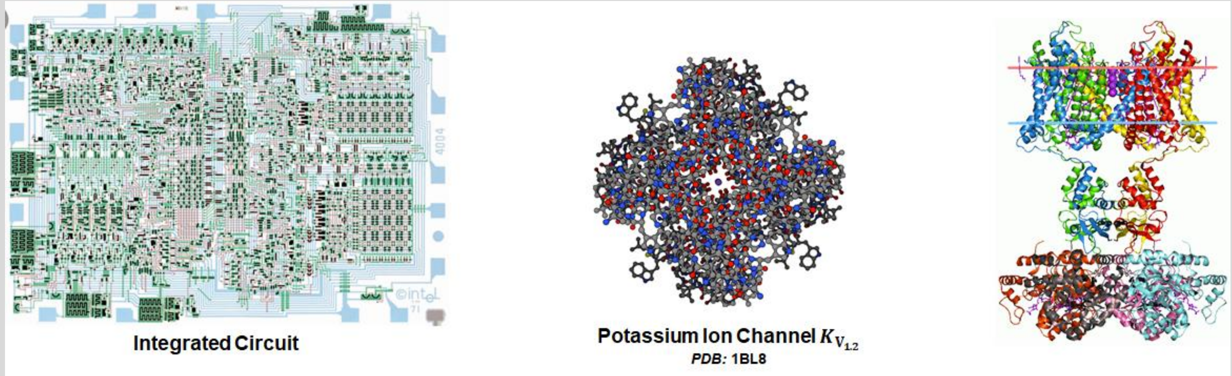
PDB: 1BL8



Without Conservation of Current

Need to Know ALL charges at all times!!

$$\mathbf{div\ J} = -\frac{\partial \rho}{\partial t}$$



Hopeless in large systems !

Conservation of Total Current is an <u>Enormous Simplification</u> because

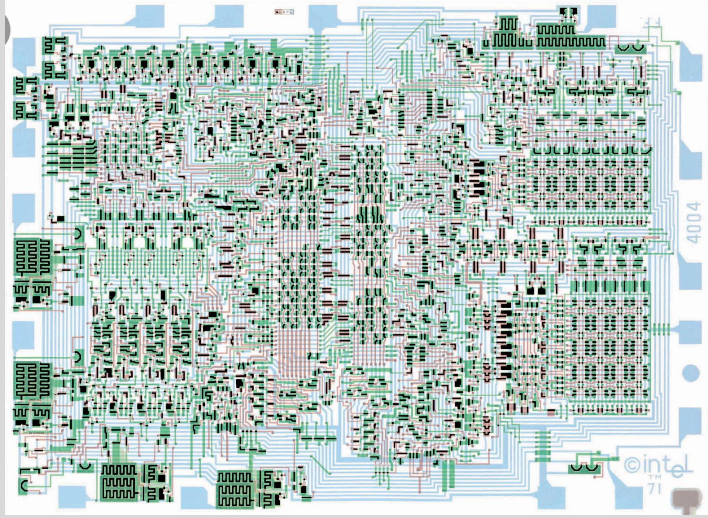
Location of charges is not needed.

Location of charges is rarely used in the design of semiconductor circuits, even at 10^{-10}sec.

$$\mathbf{div}\left(\overbrace{\mathbf{J}+\ \varepsilon_0\,\partial\mathbf{E}/\partial t}^{Total\ Current}\right)=0$$

Flux of all charge with mass

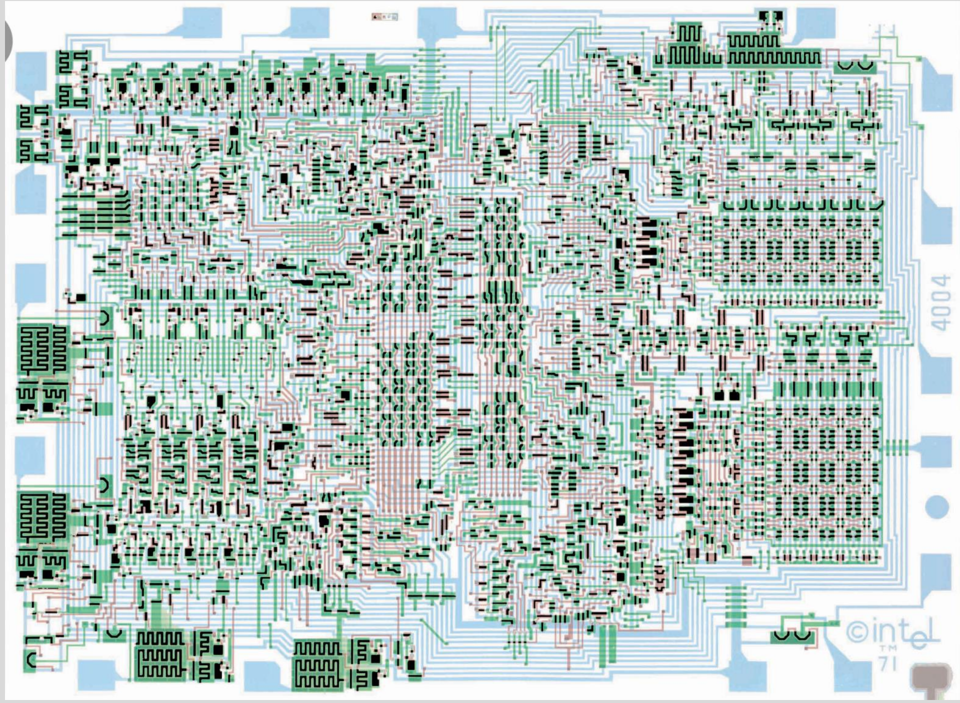
Relativistic
Property of Space
NOT matter



Do not need to know location of charges in circuit applications!

Any Questions?

Applications To Semiconductor Devices



Conservation of Total Current

is

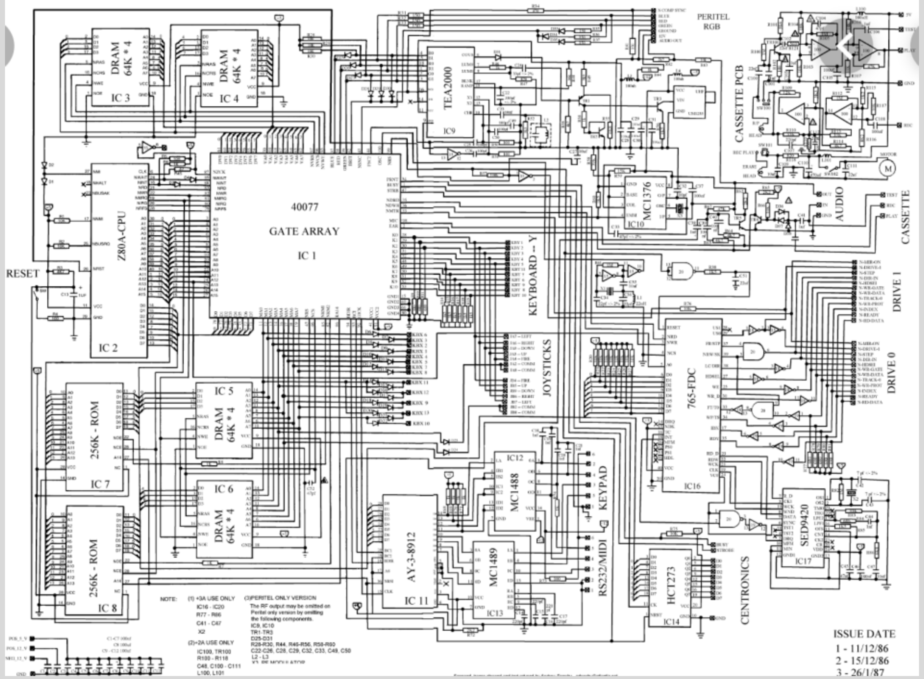
an Enormous Simplification

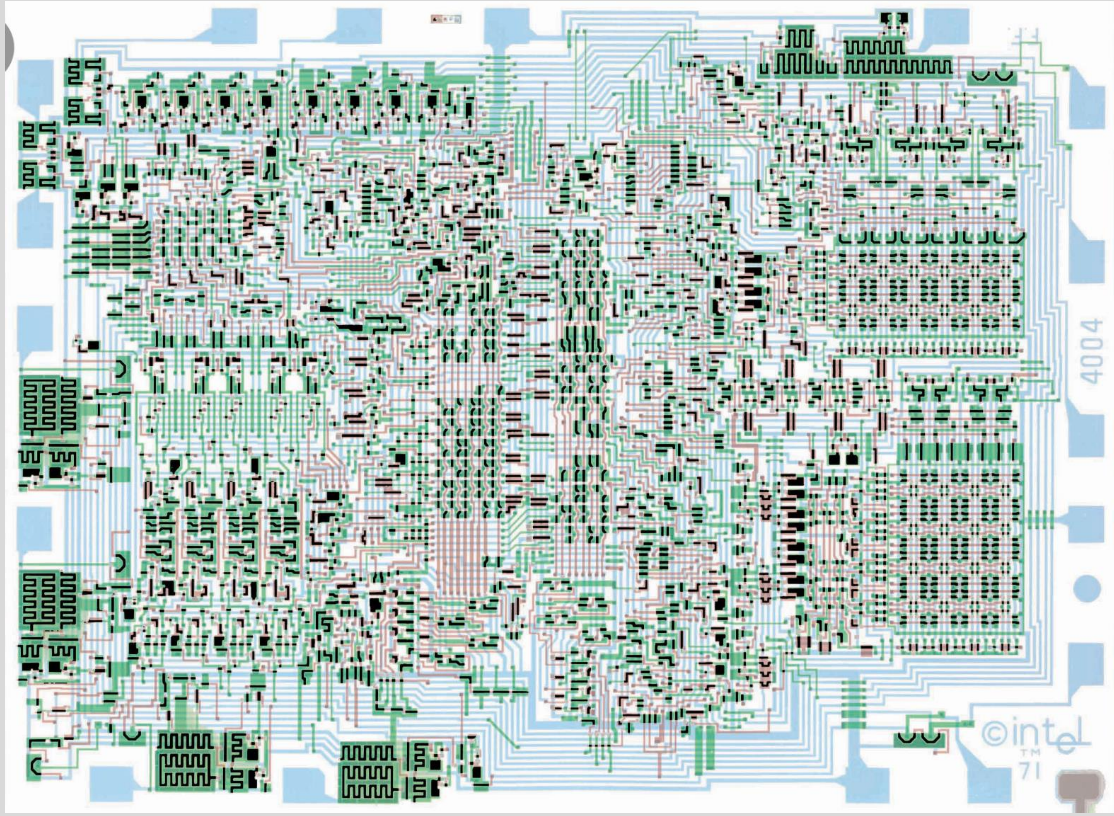
Allows an exact Kirchhoff Current Law

Eisenberg, Gold, Song, and Huang. 2018. What Current Flows Through a Resistor? arXiv preprint arXiv:1805.04814.
Eisenberg, R. S. 2019. Kirchhoff's Law can be Exact. arXiv preprint available at https://arxiv.org/abs/1905.13574.

Integrated Circuits are Designed with Kirchhoff’s Current Law and little else!!!

Hard as that is to believe





Even at 10^{-10} sec

Source: textbooks and internet

How can that possibly be?

Usual Derivation of Kirchhoff's law

is on

Long Time Scale $\sim 10^{-6}$sec

NOT AT ALL at 10^{-10} sec

How can that possibly be?

Usual Derivation of Kirchhoff's law is about fluxes

BUT

FLUXES ARE NOT CONSERVED

according to experiment or Maxwell equations

NOT AT ALL at 10^{-10} sec

How can that possibly be?

Maxwell div $\mathbf{J}_{total} = 0$

and

Kirchhoff div $\mathbf{J} \neq 0$

DISAGREE

Usual derivation of Kirchhoff
uses div of flux J of charges, but J is not conserved
$\mathbf{div}\,\mathbf{J} = -\,\varepsilon_0\,\partial\mathbf{E}/\partial t \neq 0$

Paradigm Change

Kirchhoff's Current Law Should be for TOTAL CURRENT

On time scale of semiconductor devices

Kirchhoff's Laws Should Describe TOTAL CURRENT Not flux

Valid whenever branched network is valid
And Maxwell's Core Equations are Valid

Eisenberg, Gold, Song, and Huang. 2018. What Current Flows Through a Resistor? arXiv preprint arXiv:1805.04814.
Eisenberg, R. S. 2019. Kirchhoff's Law can be Exact. arXiv preprint available at https://arxiv.org/abs/1905.13574.

Deriving Kirchhoff's Laws
from Maxwell Equations

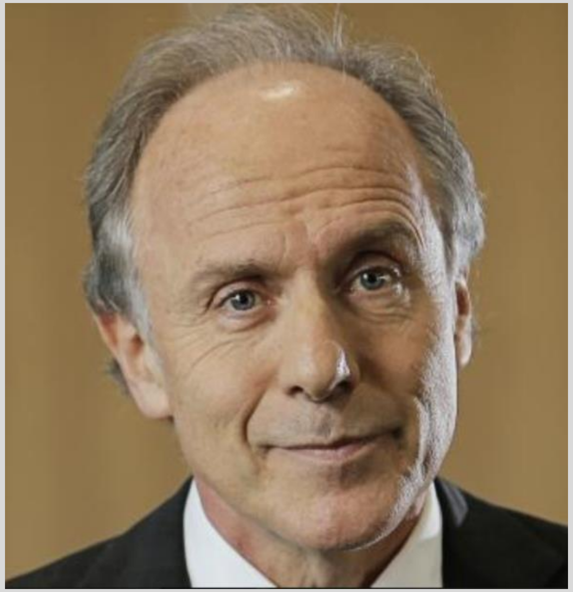
and conservation of current

is

Trivial if you use Total Current

Eisenberg, Gold, Song, and Huang. 2018. What Current Flows Through a Resistor? arXiv preprint arXiv:1805.04814.
Eisenberg, R. S. 2019. Kirchhoff's Law can be Exact. arXiv preprint available at https://arxiv.org/abs/1905.13574.

All is obvious to a fine practicing engineer and old friend



Alan Finkel
Co-designer* of AxoPatch Amplifier
Founder Axon Instruments**,**
Recently
Chief Scientist
Australian Goverment

"Bob, why do you need all that math?
Everyone knows how to use Kirchhoff.
Everyone knows you have to include the displacement current.
No one would try to keep track of all the charges"

Paraphrase of email exchange, with permission

***with Eisenberg, Levis, and Rae**

Conservation of Current

is

Important in Classical Biology

although most biologists do not know that

Conservation of Current
is
Important in Biology
in
Understanding
Transporters,
Oxidative Phosphorylation,
Photosynthesis

main processes in life

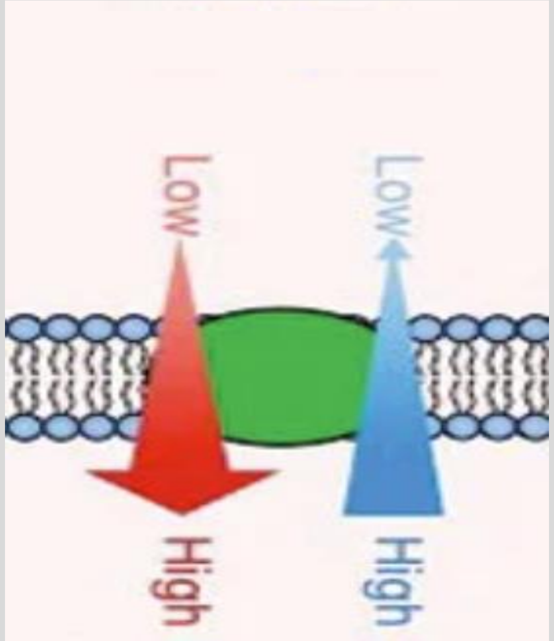
arXiv 2002.09012 (2020); DOI: 10.20944/preprints202009.0349.v2

Applying Maxwell to Transporters

Sum of Total Currents in a Transporter = zero in ‘small cell’, mitochondrion, etc.

so

Currents are Coupled in a Transporter in a ‘small cell’
by Maxwell’s equations
Conservation of Total Current



$$\mathbf{div}\left(\mathbf{J}(x,t)+\varepsilon_0\frac{\partial \mathbf{E}(x,t)}{\partial t}\right)=\mathbf{0}$$

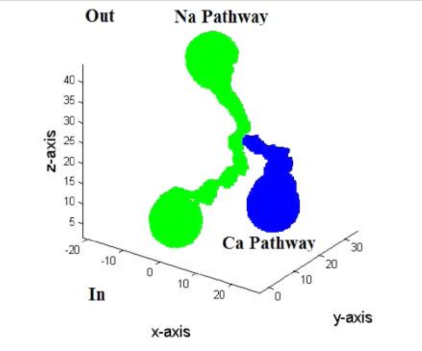
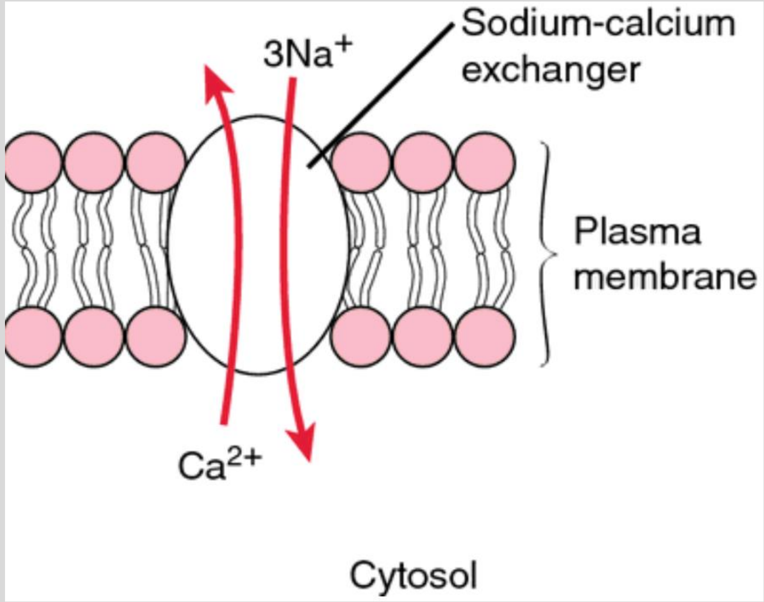


Figure 5. An inward-facing 3D model of Na^+ and Ca^{2+} pathways in NCX.



Liu, Hsieh, Eisenberg. 2016.
J Phys Chem B 120:2658-2669.

arXiv 2002.09012 (2020); DOI: 10.20944/preprints202009.0349.v2

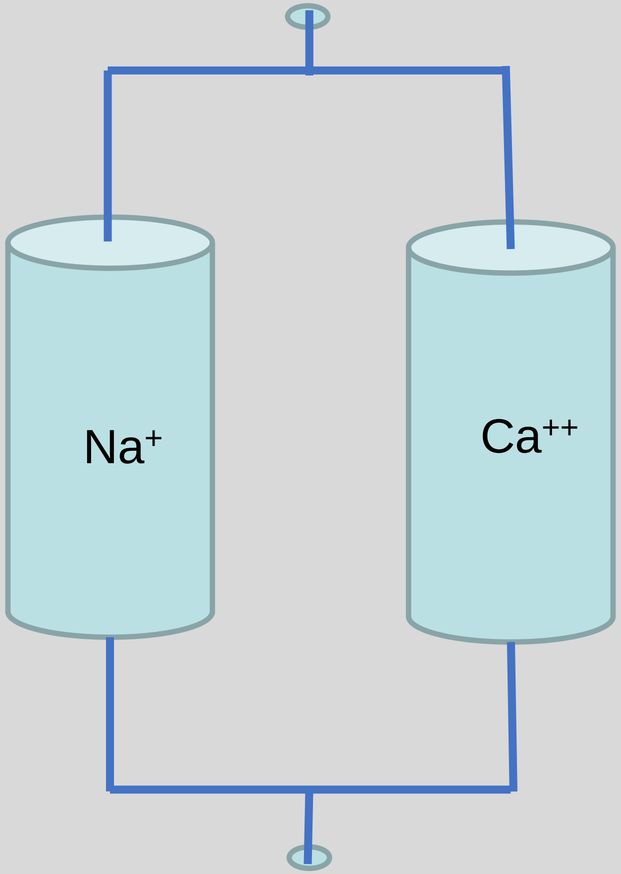
Natural Setup: small cell, etc.

Homogeneous Neumann Boundary Condition for total current
Fig. 10 of Hodgkin Huxley Katz 1952

Bilayer Setup ‘voltage clamp’

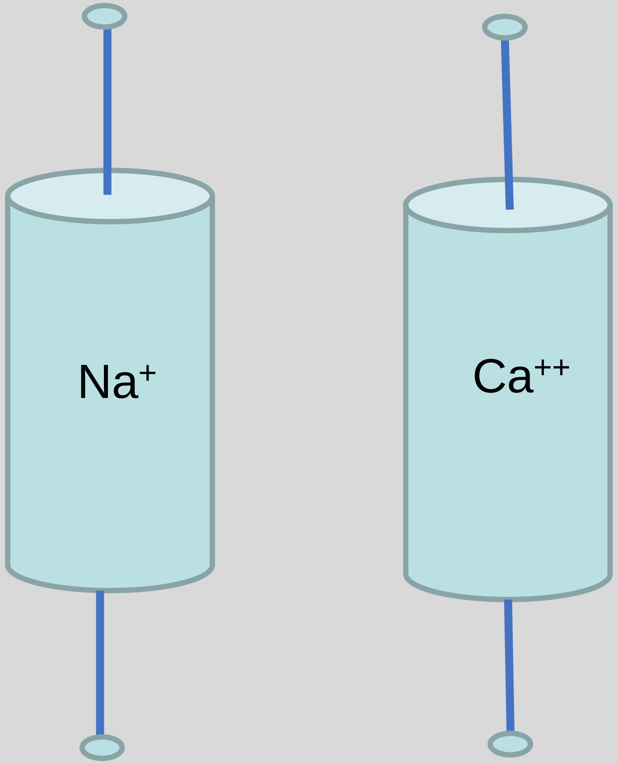
Inhomogeneous Dirichlet Condition
for Classic Voltage Clamp Hodgkin Huxley 1952

arXiv 2002.09012 (2020); DOI: 10.20944/preprints202009.0349.v2



$$I = I_{Na} + I_{Ca} = \mathbf{0}$$

$$I_{Na} = -I_{Ca}$$



$$\underbrace{I_{Na} \qquad\qquad I_{Ca}}$$

Currents *not* coupled by Maxwell
because of voltage clamp amplifier

Applying Maxwell to Transporters

arXiv 2002.09012 (2020); DOI: 10.20944/preprints202009.0349.v2

Natural Setting

1) Sum of Currents in a Transporter is zero in ‘small cell’, mitochondrion, etc.
2) Currents are Coupled in a Transporter in a natural setting by Maxwell

Experimental Setting

3) Bilayer voltage clamp up does NOT require currents to sum to zero.
Voltage clamp sets voltage across transporter; currents are not controlled.
4) So transporter currents are NOT coupled by Conservation of Current
in standard bilayer setup because of voltage clamp amplifier
but may still be coupled on atomic scale, e.g., conformation changes

Biophysical Prediction

Coupling of Transporters Depends on Setup

not just the transporter itself

Ratio of Fluxes J_{Ca}/J_{Na} is Different
in Vesicle (e.g., mitochondrion) **and Bilayer**

Paradigm Change

Biophysical Prediction
With Conservation of Current
no more difficult than
large circuit problems

Biophysical Prediction*

With Charges

is more or less

Impossible

because of the enormous number 10^{18} of charges and their interactions on atomic and macroscopic scales.

***using continuity equation, for example.**

Any Questions?

Extra Slides

Conservation of Total Current is NOT a theory of everything

Only total current is conserved

Conservation of Total Current is NOT a theory of everything

Conservation only describes **Total Current**

When flux of charge or charge movement is important, conservation of current is not enough

$$\textbf{div } \mathbf{J} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \neq 0$$

Flux J of charged matter is NOT NOT NOT conserved

$$\mathbf{div\ J} \neq \mathbf{0}$$

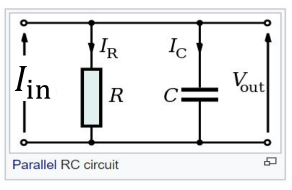
Maxwell core Equations imply that

Charge and Matter accumulate because $\varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \neq 0$

Relativistic Property of Space NOT matter

$$\mathbf{div\ J} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \neq 0$$

In physical language of electronics,
Some charge accumulates in the 'stray capacitance' of space
independent of matter



Parallel RC circuit

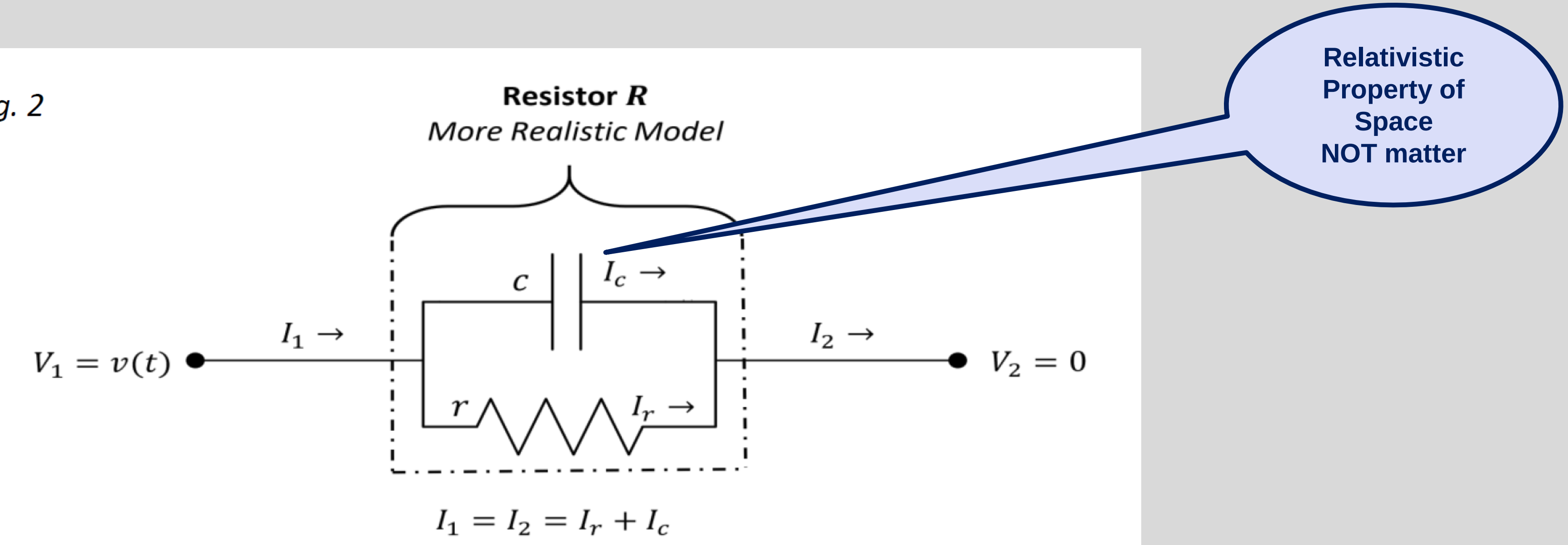
$$V_{out} = I_{in}\left(1 - e^{-t/RC}\right)$$

Eisenberg, Gold, Song, and Huang. 2018. What Current Flows Through a Resistor? arXiv:1805.04814.
Eisenberg, R. S. 2019. Kirchhoff's Law can be Exact. arXiv preprint available at arXiv:1905.13574.

Flux into a Resistor does NOT equal Flux out of Resistor

Total Current I_1 into a Resistor DOES EQUAL the Total Current I_2 out of a Resistor

Fig. 2



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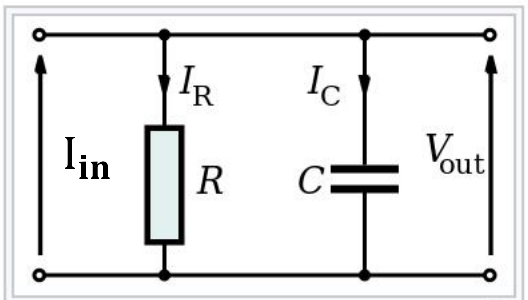


Parallel RC circuit

$$V_{out} = I_{in}\left(1 - e^{-t/RC}\right)$$

$RC =$ charging time $= 10^{-12}$ farads $\times\ 10^{3}$ ohm = 10^{-9} sec

FLUXES ARE NOT CONSERVED

Eisenberg, Gold, Song, and Huang. 2018. What Current Flows Through a Resistor? arXiv preprint arXiv:1805.04814.
Eisenberg, R. S. 2019. Kirchhoff's Law can be Exact. arXiv preprint available at https://arxiv.org/abs/1905.13574.

Corollaries of Maxwell's Core Equations

Continuity Equation

Linking Flux and Content

Corollaries of Maxwell's Core Equations

Derivation of the Continuity Equation

Linking Flux and Content

$$\mathbf{curl}\ \ \mathbf{B} = \mu_0 \left(\overbrace{\mathbf{J}(x,t)}^{\text{Flux of All Charges}} + \varepsilon_0\, \partial\mathbf{E}/\partial t \right)$$

$$\mathbf{div\ curl}\ \ \mathbf{B} = 0 = \mu_0\, \mathbf{div}(\mathbf{J} + \varepsilon_0\, \partial\mathbf{E}/\partial t)$$

$$\mathbf{div\ J} = -\,\varepsilon_0\ \mathbf{div}\ (\partial\mathbf{E}/\partial t) = -\ \varepsilon_0\, \boldsymbol{\partial}(\mathbf{div\ E})/\boldsymbol{\partial t}$$

But $\mathbf{div\ E} = \rho/\varepsilon_0$

$$\boxed{\mathbf{div\ J} = -\,\partial\rho/\partial t}$$

Hopeless, if one must

"… exhibit in every case all the charges, whatever their origin" at all times

Section 10-4 of Feynman, Leighton, and Sands (1963) vol. 2 ***Electromagnetism and Matter***

Conservation of Total Current

$$\mathbf{div\ J_{total} = 0}$$

Is enormously helpful in Applications

as well as scary

Continuity Equation

Linking Flux and Content

Feynman's Hidden Implications

$$\mathbf{div\ J} = -\frac{\partial \rho(x, y, z|t)}{\partial t}$$

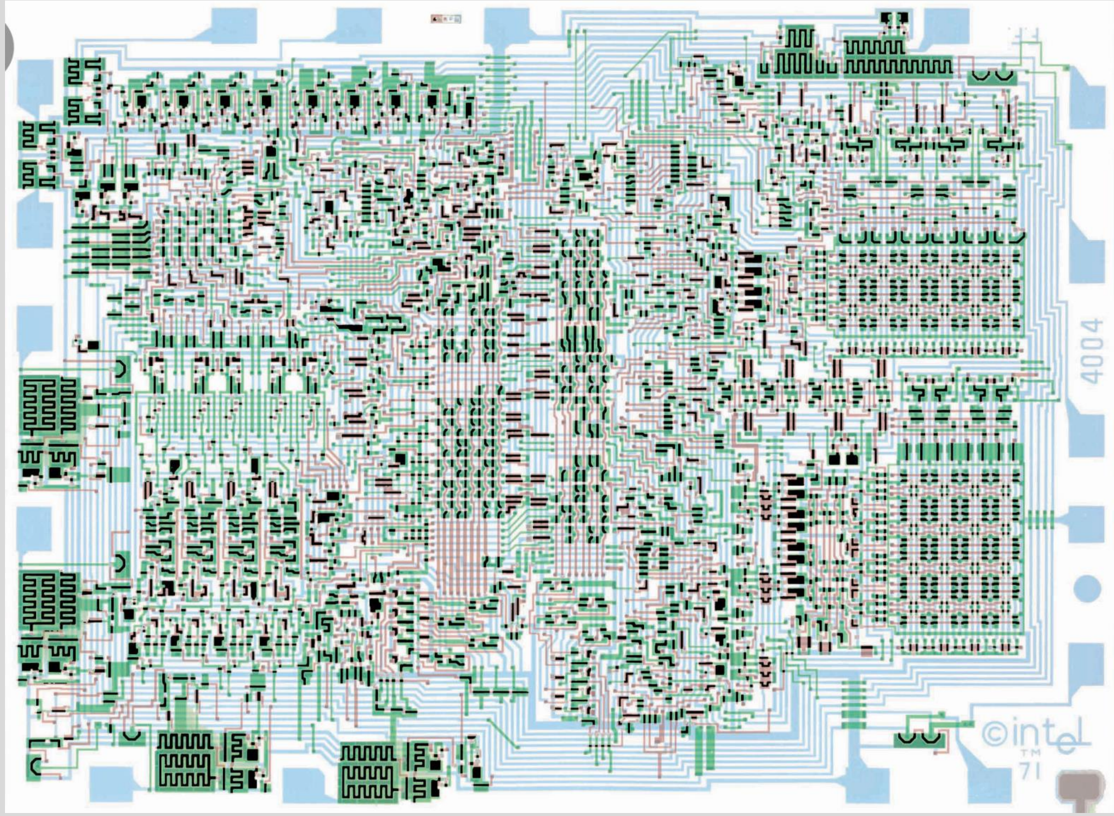
Oh Dear!

Often impossible

Must know all charges $\rho(x, y, z|t)$

at all times

integrated circuit



Source: textbooks and internet

Seems Hopeless

Fortunately, it is not hopeless

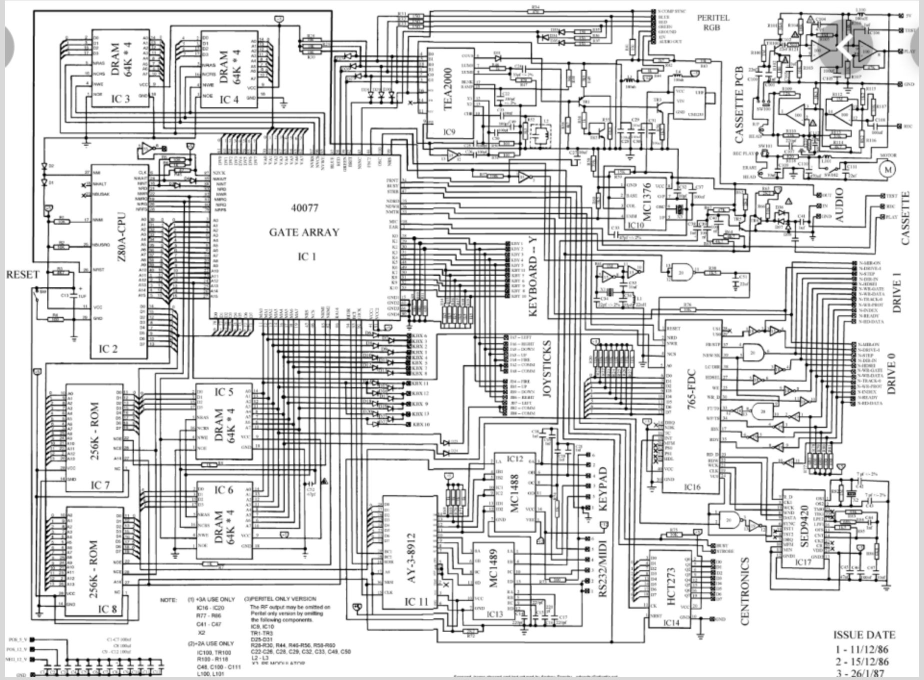
Stay Tuned

Kirchhoff’s Current Law
Brings hope

It is NOT necessary in our computers to know all the charges!

Kirchhoff’s Current law is (almost) **enough**

It is NOT necessary in our computers to know all the charges! Kirchhoff’s law is (almost) enough



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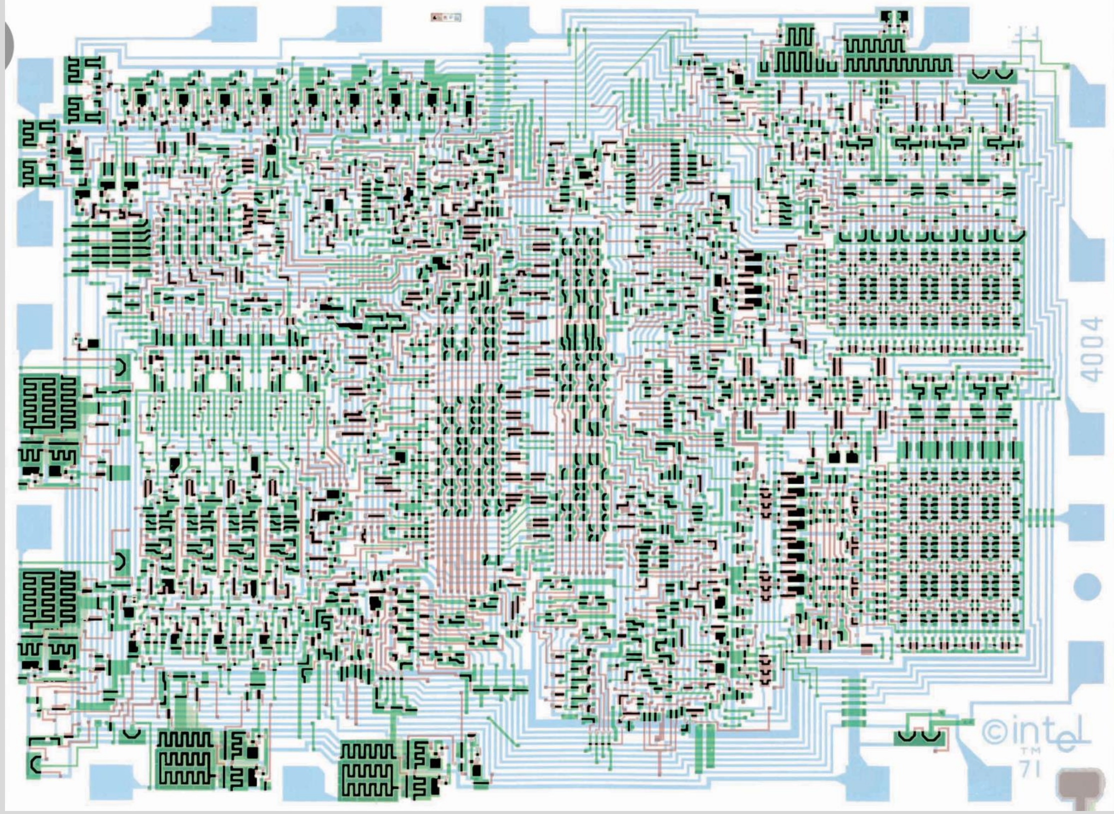

Source: textbooks and internet

Without Conservation of Current
Need to Know ALL charges at all times!!

Hopeless in large systems

where all ions interact with each other!

Inside Channels
PROFOUND SIMIPLIFICATION
If we can figure out how to exploit it

Profound Implications of One Dimensional Systems for atomic view of ion channels

Current is equal everywhere in a channel
At all times and under all conditions
that the Maxwell Equations Apply

Maxwell's Core Equations

Describe Electricity with no errors, $< 10^{-6}$

almost everywhere at any time

Relativistic Property of Space NOT matter

$$\mathrm{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \mathrm{div}\,\mathbf{B} = 0$$

$$\mathrm{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \mathrm{curl}\,\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

Feynman's 'Implicit Assumptions' are ONLY in ρ and J

Conservation of Total Current

is important in

Biology

Paradigm Change

View of Channels has been focused on movements of individual ions in channels,

But

Total Current flow is equal everywhere in a one dimensional channel

Paradigm Change

Position does <u>not</u> appear in equations for current flow in a one dimensional channel