# What is Total Current?

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**June 21, 2023**

*‘Total Current’ is defined by eq. 3. Section 328, Vol. 1*
*J.C. Maxwell (1865) “A Treatise on Electricity and Magnetism”*

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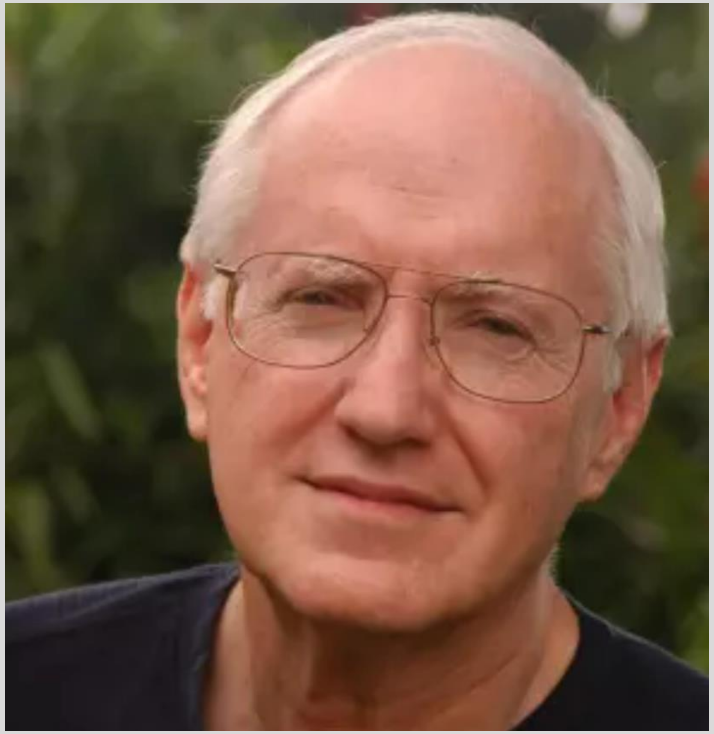
Total Current is defined by Maxwell as the source of the magnetic field, in his version of Ampere's law. Ampere's law in a vacuum defines radiation only if the displacement current is included in the definition of total current. Maxwell says that "the true electric current that on which the electromagnetic phenomena depend, is not the same thing as the current of conduction, but that the time-variation of the electric displacement, must be taken into account in estimating the total movement of electricity, …."

Maxwell's statement is unambiguous, but it is overlooked in almost all textbooks of electrodynamics since roughly 1905. Overlooking displacement current has serious consequences for modern electronics. The ignored displacement current is important in modern circuits. It is particularly large in digital circuits in which the fundament bits and bytes last only a handful of nanoseconds. The displacement current is described by a differential equation in time. Ignoring the derivative in time means losing the boundary condition in time, the initial condition that establishes the charges in the circuit that drive the steady state current. If charge is ignored, paradoxes and contradictions arise when considering circuits. Maxwell himself analyzed stratified conductors and dielectrics and discussed at length the transients that arise from the time derivative of displacement current. If only conduction current is used in the analysis, these time derivatives disappear. Total current accounts for the transients prominent in experiments. Maxwell shows that the model of conduction must be changed to include *ad hoc* capacitors—with values determined phenomenologically—to produce the transients found in experiments, if displacement currents are ignored.

Kirchhoff's law can be rewritten to describe total current instead of conduction current and it then is identical to Maxwell's treatment of stratified dielectrics. Electrodynamics should use the total current defined by Maxwell if it wishes to describe experimental results, and be true in that sense.

Thanks to Xavier Oriols for inviting me to speak at

# Special Session in Honor of Dave Ferry



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# Total Current

is a

# Central Idea

## in Classical Electrodynamics

*since ~1865*

**that is overlooked in modern textbooks**

# Total Current is a Central Idea

**in Classical Electrodynamics**

## Maxwell says:

"One of the chief peculiarities of this treatise is the doctrine which it asserts, that

**the true electric current**

that on which the electromagnetic phenomena depend,

**is not the same thing as the current of conduction,**

but that the

**time-variation of the electric displacement, must be taken into account**

in estimating the total movement of electricity, …."

***Section 610, p. 233, Vol 2***

***Maxwell, J. C. (1865). A Treatise on Electricity and Magnetism*** *(reprinted 1954).*

***I will try to convince you:***

# Kirchhoff's law should use TOTAL current

**that includes displacement current $\varepsilon_0 \partial \mathrm{E}/\partial t$**

*as Maxwell did p. 376, Section 328 "A Treatise on Electricity and Magnetism"*

**Kirchhoff's Law says**
**"What flows in, flows out,**
**Without accumulation."**

**but, according to experiments, and Maxwell,**
**electrons accumulate!**

## What is Current?

# Total Current was Defined by Maxwell* as the Source§ of the Magnetic field

** in Equation 3, Section 328 "A Treatise on Electricity and Magnetism" (1865, reprinted 1954)*

*§ right hand side of the Maxwell Ampere Law:* $\mathbf{curl}\ \mathbf{B} = \mu_0\ \mathbf{J}_{total}$

*see line 245 J.C. Maxwell, 1864 Dynamical Theory of the Electric Field, on p. 260 & 297 of Simpson (1998) Maxwell on the Electromagnetic Field: A Guided Study, Rutgers*

# Total Current is NOT the Flow of Charge

## How do we know that?

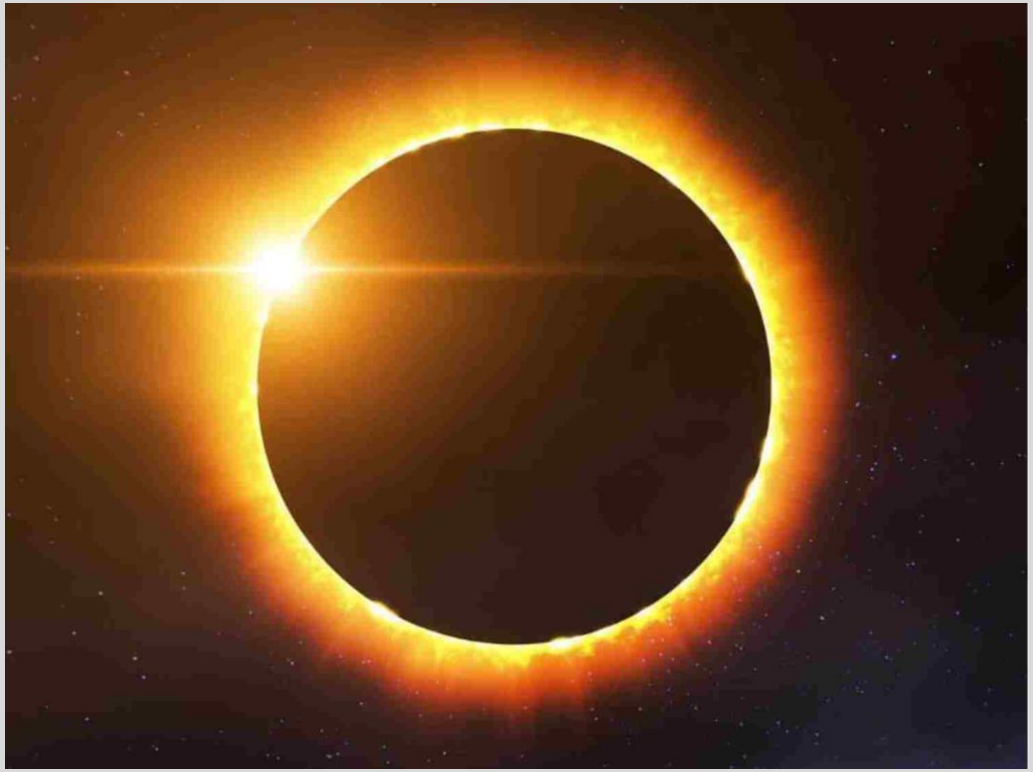
# **Magnetic Fields Exist in Vacuum**

**Charge and Flow of Charge are ZERO in a vacuum**

**Magnetic Fields in Vacuum Create Electromagnetic Waves**

***LIGHT***

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0 \qquad c = 1/\sqrt{\varepsilon_0 \mu_0} = \text{velocity of light} \qquad \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$



# Light travels through the Vacuum of Space

## ethereal current $\varepsilon_0 \partial \mathrm{E}/\partial t$ flows in vacuum of space,

### once thought to be filled with an ‘aether’

Maxwell, 1865. Treatise on Electricity and Magnetism
Jeans, 1908. The Mathematical Theory of Electricity and Magnetism.
Whittaker, 1951. A History of the Theories of Aether & Electricity.
Simpson, 1998. Maxwell on the Electromagnetic Field: A Guided Study.

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

**Wave Equation**
Corollary of
Maxwell Equations

$c = 1/\sqrt{\varepsilon_0 \mu_0}$ = *velocity of light*

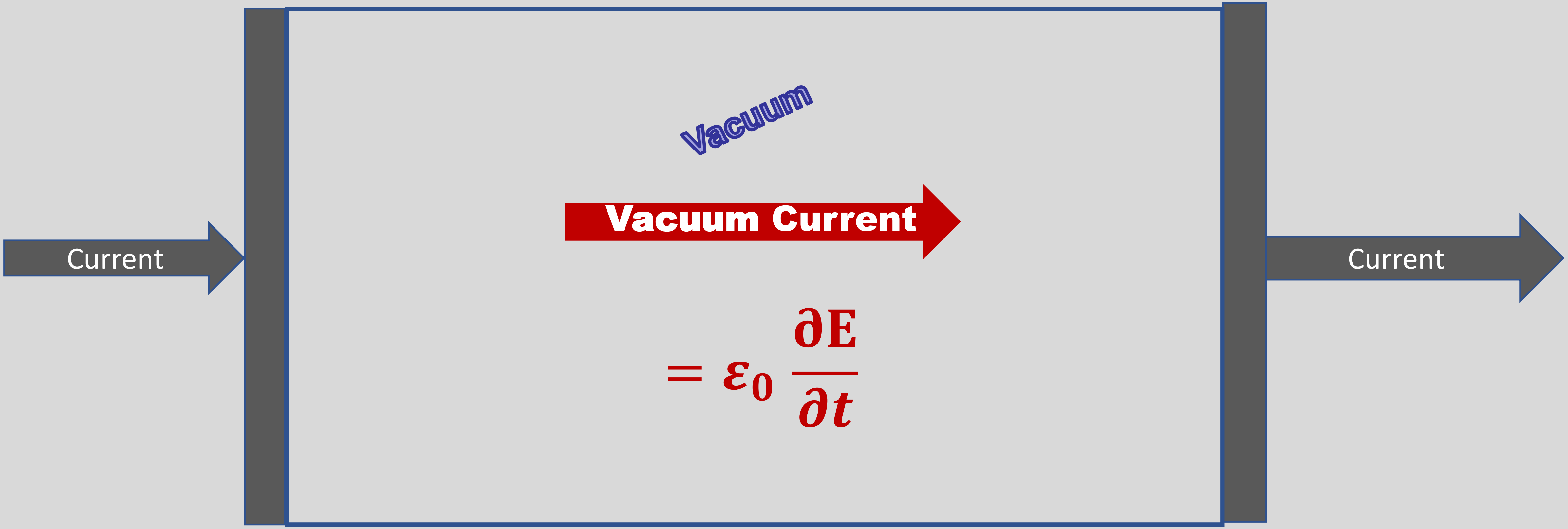
$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

# Well known Example

taught,

*or should be taught,*

In First Year of Physics




$$= \varepsilon_0 \frac{\partial E}{\partial t}$$

**Vacuum current = Ethereal current = Displacement Current**
**All are names for the same thing $\varepsilon_0\, \partial E/\partial t$**

# Maxwell Ampere Law in a Vacuum

Magnetism

$$\frac{1}{\mu_0}\,\text{curl}\,\overset{\downarrow}{\mathbf{B}} = \underbrace{\varepsilon_0\,\frac{\partial \mathbf{E}}{\partial t}}_{\text{Total Current}}$$

*Vacuum contains no charge*
*and thus flow of charge* $\mathbf{J} = 0$

**No known error between stars, inside atoms on all time scales, even those of thermal motion**

# Maxwell Ampere Law in Matter

*No known error, inside atoms or between stars*

Magnetism ↓

Universal Displacement Current

$$\frac{1}{\mu_0}\,\text{curl}\,\mathbf{B} = \mathbf{J} + \underbrace{\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}} = \mathbf{J}_{total}$$

***Fundamental <u>Experimental</u> Result***
*in Physics*
**$\varepsilon_0 \partial \mathbf{E}/\partial t$ exists everywhere,**
*e.g., inside atoms*

**J is the flow of all charge,**
*however brief, small or transient.*
**J includes polarization charge of dielectrics**

# Maxwell-Ampere Equation

$$\frac{1}{\mu_0}\,\mathbf{curl\ B} = \mathbf{J}_{total} \qquad \textbf{(1)}$$

$\mathbf{J}_{total}$ includes the flux of charge with mass, however brief, small, or transient.
$\mathbf{J}_{total}$ includes the polarization charge of dielectrics.

# Identity

$$\mathbf{div\ curl}\ \mathit{anything} = \mathbf{0} \qquad \textbf{(2)}$$

**Equations imply, by mathematics alone,**

**Kirchhoff Current Law for Fields**

$$\mathbf{div\ J}_{total} = \mathbf{div\ curl\ B} = \mathbf{0}$$

Kirchhoff Current Law for Fields

$$\mathbf{div}\ \mathbf{J}_{\boldsymbol{total}} = \mathbf{div}\ \mathbf{curl}\ \mathbf{B} = \mathbf{0}$$

**<u>Maxwell-Ampere Equation</u>**

*implies*

**<u>Zero Accumulation of Total Current</u>**

$\mathbf{J}_{\boldsymbol{total}}$ includes the flux of charge with mass, however brief, small, or transient.
$\mathbf{J}_{\boldsymbol{total}}$ includes the polarization charge of dielectrics.

Maxwell → $\text{div } \mathbf{J}_{total} = 0$

and

Classical Kirchhoff → $\text{div } \mathbf{J} = 0$

**DISAGREE**

$$\mathbf{J}_{total} = \mathbf{J} + \varepsilon_0 \, \partial \mathbf{E}/\partial t$$

**Usual derivation of Circuit Kirchhoff Discusses only flux J of charges**

**Derivation should discuss $\mathbf{J}_{total}$**

# The flux of charges accumulates

as 'free' charge $\rho$

by Continuity Equation $\mathbf{div\,J} = -\varepsilon_0\, \partial\rho/\partial t \quad \Rightarrow \quad \mathbf{div\,J \neq 0}$

**J does accumulate** $\qquad$ $\mathbf{J}_{total}$ **does not accumulate,**

**If** $\mathbf{div}\,(\mathbf{J} + \boldsymbol{\varepsilon_0}\,\partial\mathbf{E}/\partial \boldsymbol{t}) = \mathbf{0}, \quad \mathbf{div\,J \neq 0}$

**Kirchhoff's Law should**
**Be about TOTAL current**

**Kirchhoff’s Law says**
**“What flows in, flows out,**
**Without accumulation.”**

**but, as Maxwell and experiments show,**
**electrons accumulate !**

***Conclusion***
**Kirchhoff’s law should use TOTAL current**

**Paradigm Change**

# Maxwell analyzes ‘stratified dielectrics’ using <u>total current</u> in Ohm’s law

*p. 376, Section 328 “A Treatise on Electricity and Magnetism” (1865, reprinted 1954)*

**Maxwell’s transients <u>do not occur</u> when flux of electrons is used in Ohm’s law.**

***Conclusion***

**Kirchhoff’s law must use TOTAL current to agree with experiments**

**Paradigm Change**

**Maxwell's transients do not occur**
**when flux of electrons is used in Ohm's law.**

# Conclusion

**Textbook treatment of Kirchhoff's Law does not agree with experiments**
**unless the underlying model of conduction is changed**

**Paradigm Change is Needed**
**to agree with experiments**

# Important Applications

in Engineering

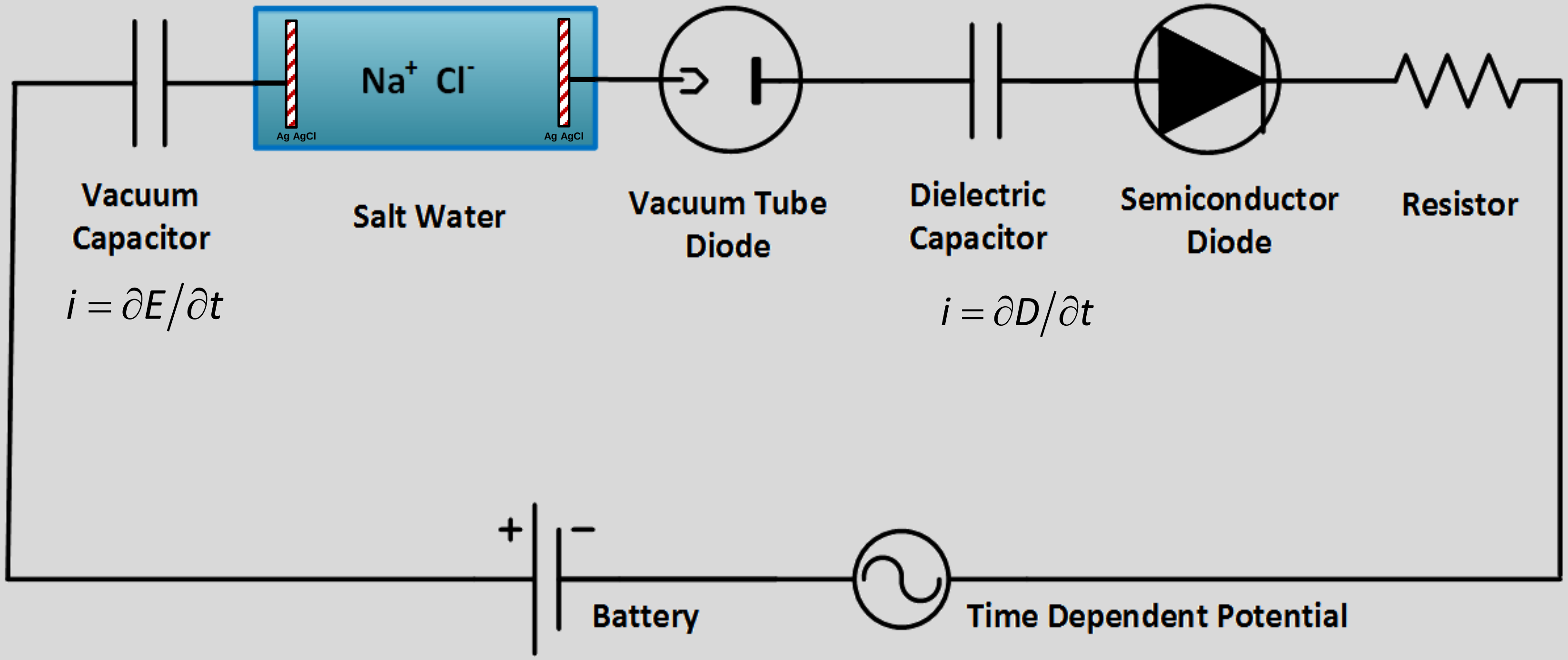
**like Circuit Analysis**
**including computer programs *Spice* and *Circuit Board Design***

# should use TOTAL current

# *Circuits are Special:*

*TRIVIAL for engineers and PROFOUND for everyone else*

**Total Current is the Same in Series Circuits and Systems**
**<u>Independent of Mechanism</u> of Charge Movement**
*Kirchhoff Coupling*




*Eisenberg (2016) Mass Action and Conservation of Current. Hungarian Journal of Industry and Chemistry 2016 44:1-28*
*also arXiv:1502.07251 44:1-28.*

***Circuits are Special:***
***TRIVIAL for engineers and PROFOUND for everyone else***

# Total Current is the Same in Series Circuits
# <u>Independent of Mechanism</u> of Charge Movement

Equality of Total Current in Series Circuit is

**NOT**

**a Property of the Local Mechanism**

of charge movement.

Equality of Total Current Depends on

**Global Properties of Electricity**,

*that I call Kirchhoff Coupling*

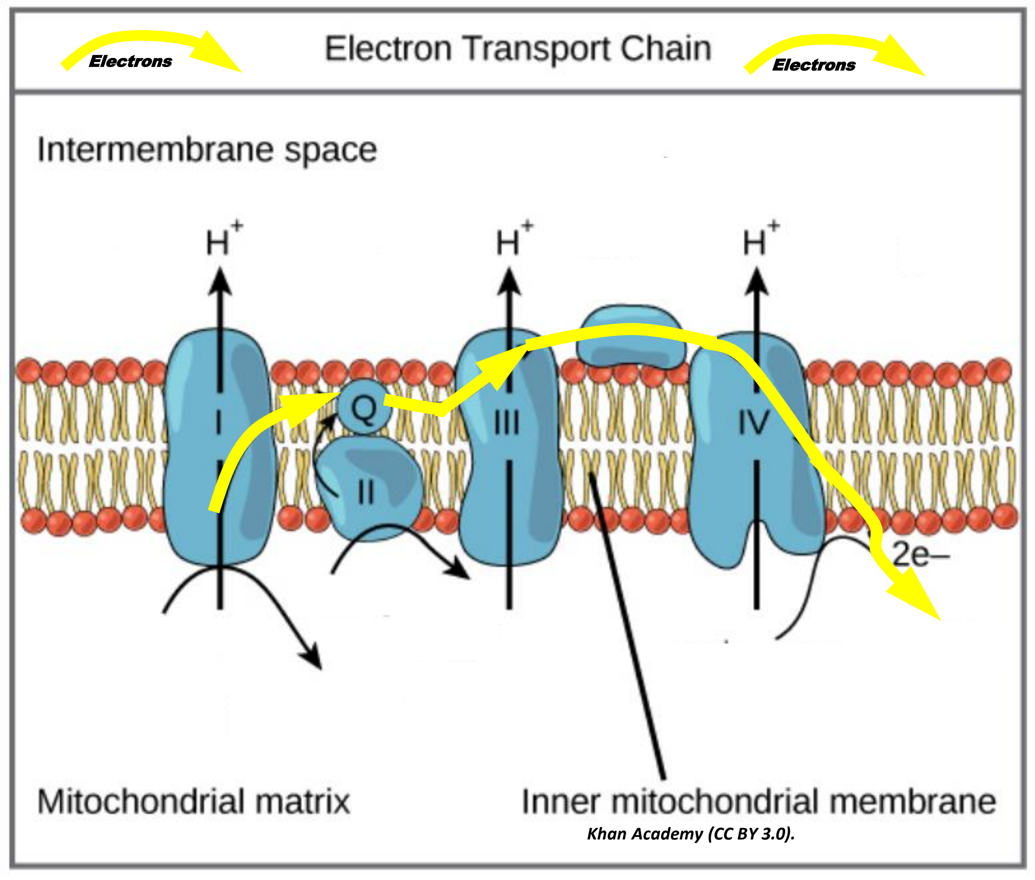
**NOT on the local mechanism of charge movement**

# Important Applications in Biology

## using Circuits and Total Current in Kirchhoff’s Law

ATP stores Chemical Energy in Biology

# Electron Flow in ATP Production



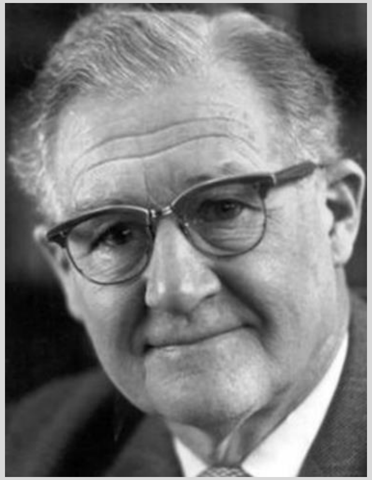

**Biologists have tried to compute Electron flow from Coulomb's law with little success since Nobel Prize to Peter Mitchell for Chemiosmotic Hypothesis 1978**

**In my view, Kirchhoff's law should be used.**

**We have started that process**

Xu,. Eisenberg, Song and Huang (2022). "Mathematical Model for Chemical Reactions in Electrolyte Applied to Cytochrome c Oxidase: an **Electro-osmotic Approach**." arXiv preprint arXiv:2207.02215.

# *Some biologists have been Applying Maxwell and Circuits to the Nerve Signal for a long time*



Alan Hodgkin William Rushton

Proc Roy Soc (London) Ser B. 1946;133:444-79.

# Channels are Chemically and Structurally INDEPENDENT

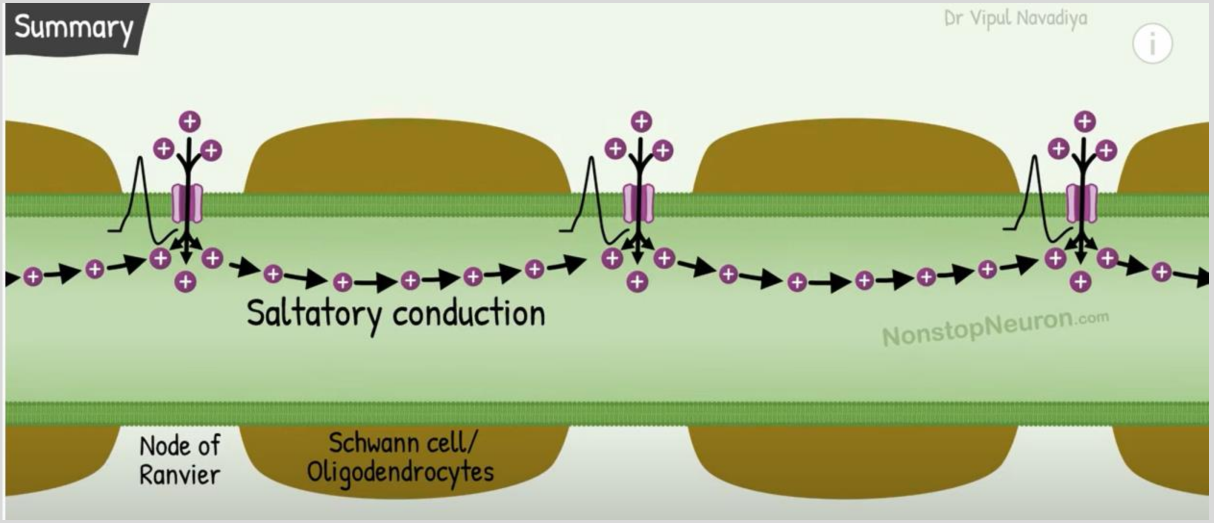
*much the way transistors are independent*

# Natural Function of Channels Requires Coupling by the Electric Field

**Electric Field Couples Channels so they can make a Useful Electrical Signal the Action Potential**

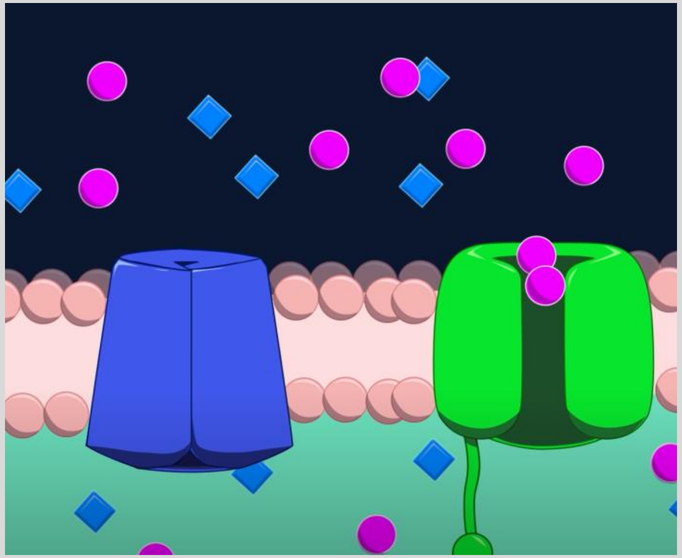
# *Coupling in Natural Function is by Electric Field, i.e., VOLTAGE SPREAD*

**Nerve Fiber with Insulating Myelin**




***https://www.youtube.com/watch?v=tOTYO5WrXFU***

**Channels**



**Na** **K**

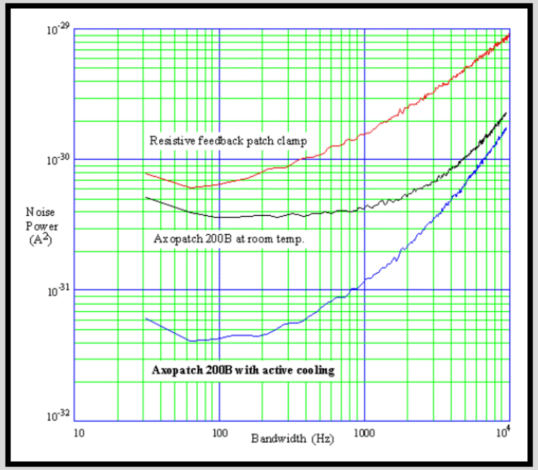
***https://www.youtube.com/watch?v=oa6rvUJlg7o***

# Our* Axopatch makes Voltage Clamp seem natural

# Clamp is Not Natural

# UNclamp is Natural !

AxoPatch 200B

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3. AMPA receptors--another twist? *Science*
4. Molecular Basis of Calcium Signaling in Lymphocytes: STIM and ORAI. *Annu Rev Immunol*
5. Neurological Channelopathies. *Annu Rev Neurosci*
6. New antiarrhythmic drugs for treatment of atrial fibrillation. *Lancet*
7. A Glial Signal Consisting of Gliomedin and NrCAM Clusters Axonal Na(+) Channels during the Formation of Nodes of Ranvier. *Neuron*
8. Small Molecule Activators of TRPML3. *Chem Biol*
9. Truncated {beta}-amyloid peptide channels provide an alternative mechanism for Alzheimer's Disease and Down syndrome. *Proc Natl Acad Sci U S A*
10. Modelling the molecular mechanisms of synaptic plasticity using systems biology approaches. *Nat Rev Neurosci*
11. Pathophysiological roles of transient receptor potential channels in glial cells. *Yakugaku Zasshi*
12. Targeted Delivery of siRNA to Macrophages for Anti-inflammatory Treatment. *Mol Ther*
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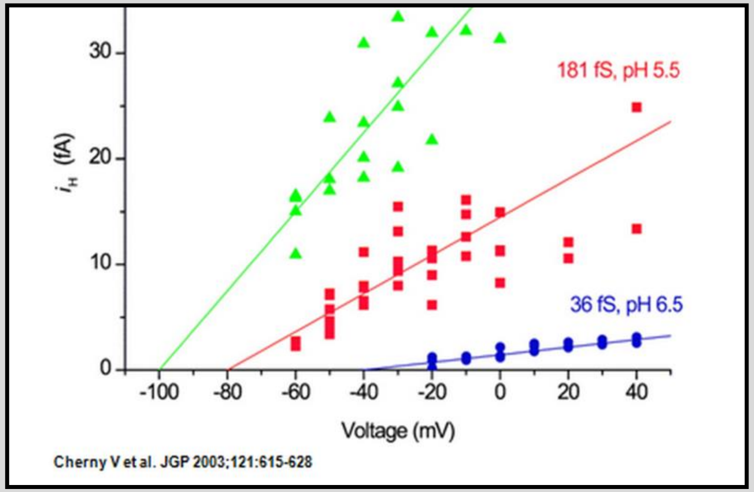



*Eisenberg, Levis, Finkel, ~1983

# Any Questions?

# Displacement Current $\varepsilon_0 \partial \mathbf{E}/\partial t$ has a Special Place in Physics

**Charge has a Special Place in Physics**

Displacement Current $\boldsymbol{\varepsilon_0}\ \boldsymbol{\partial}\mathbf{E}/\boldsymbol{\partial t}$ makes charge Lorentz Invariant

# Charge has a Special Place in physics

Charge on a particle (coulombs) does not vary

as it moves at velocity $v$,

even near the velocity of light $c$

Charge (coulombs) is “Lorentz Invariant”

Distance, time, and relativistic mass

**Change Dramatically**

near the velocity of light as $\sqrt{1 - v^2/c^2}$

# Displacement Current $\varepsilon_0 \partial E/\partial t$ is a Property of Space according to Theory of Relativity

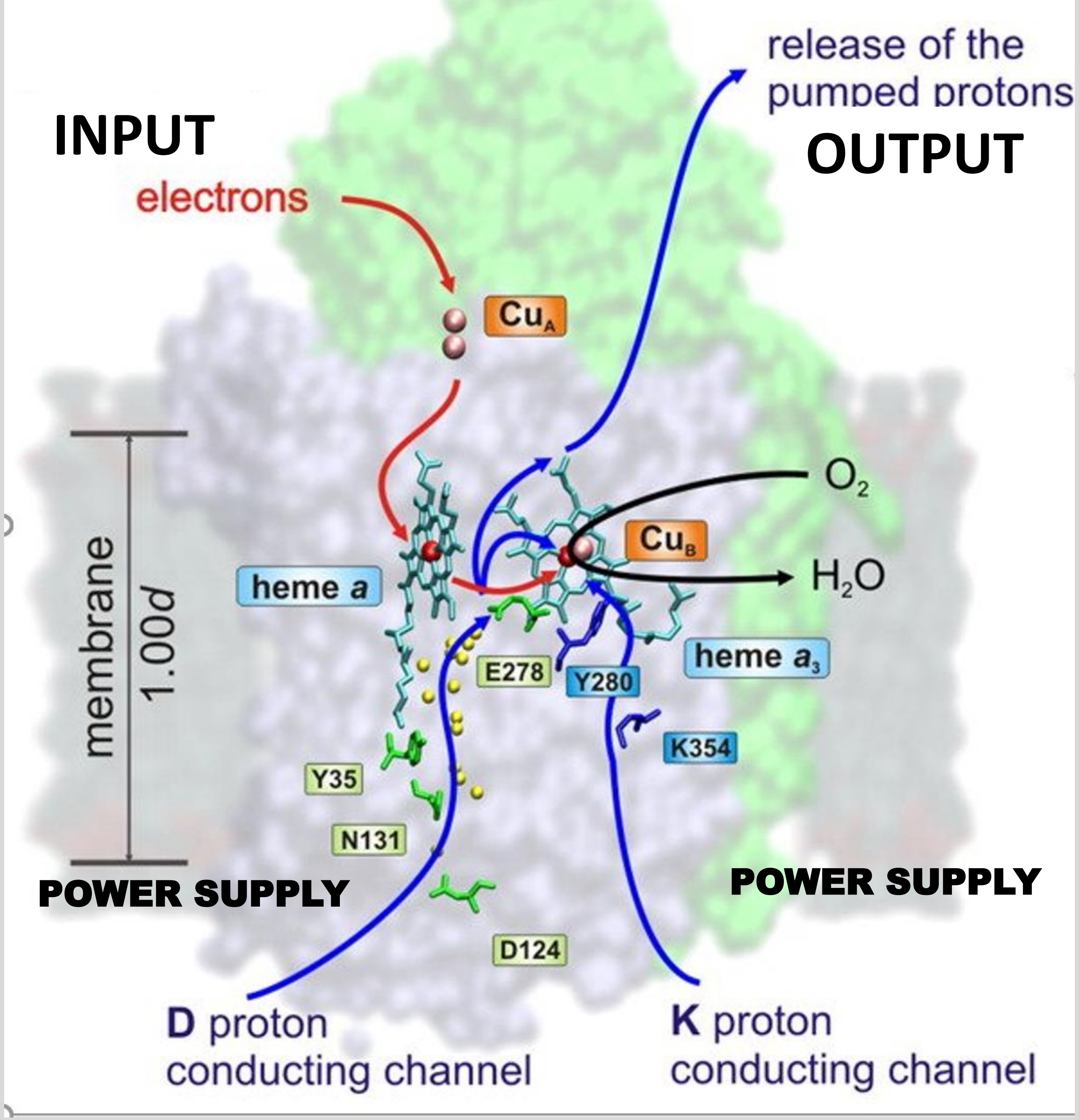
## Maxwell Equations and Relativity are almost the same thing

"**The special theory of relativity … was simply a systematic development of the electrodynamics of Clerk Maxwell and Lorentz**".

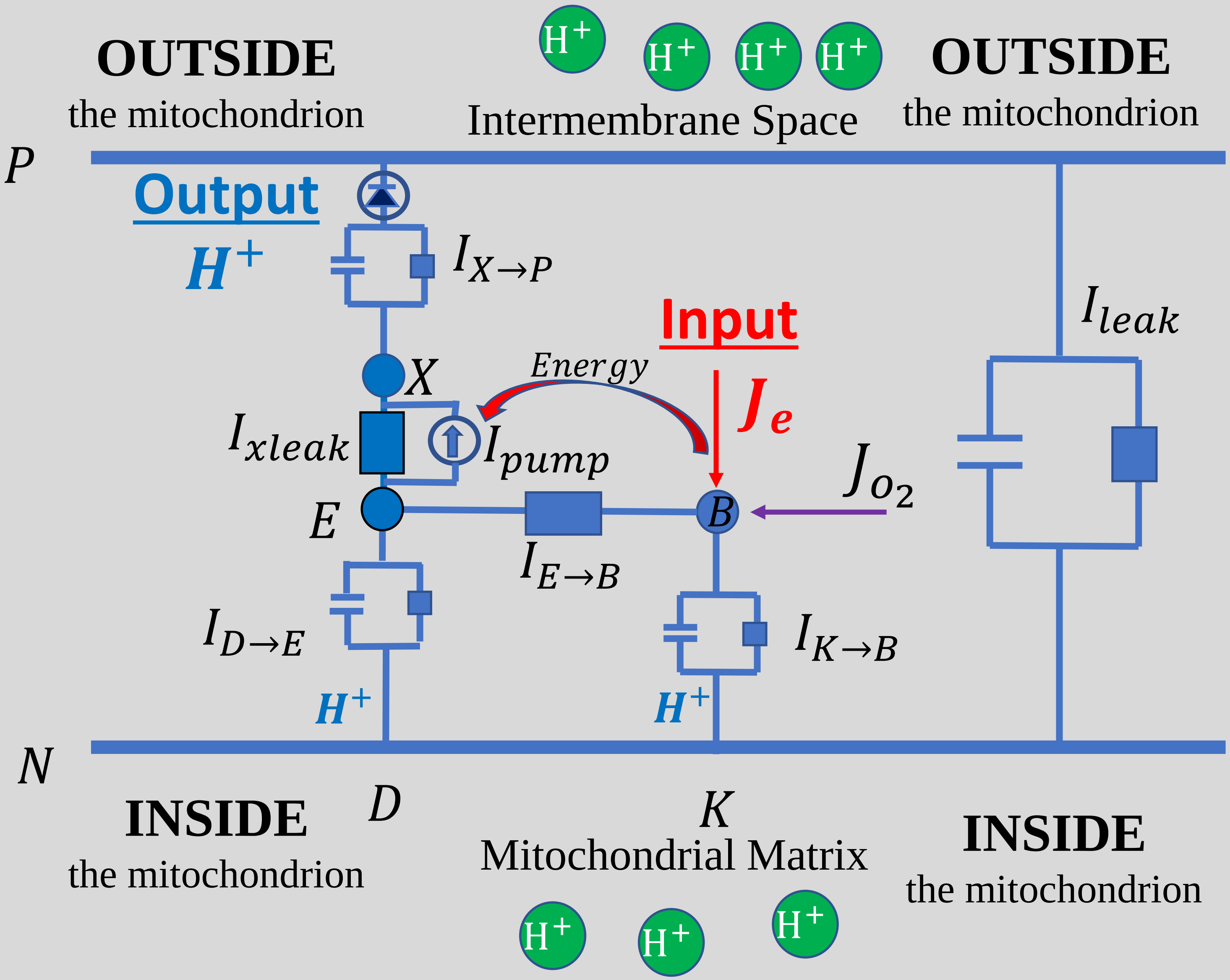
p. 57 of Einstein, A. 1934. *Essays in science, originally published as Mein Weltbild 1933, translated from the German by Alan Harris. Open Road Media.*

# Important Applications
## in Biology

Belevich et al, PNAS 2010;107:18469



# Circuit Model of Cytochrome Oxidase C



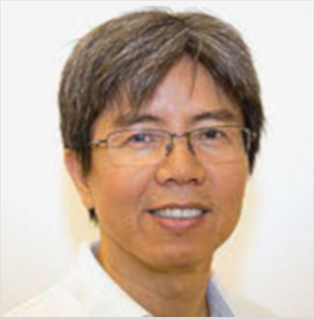

Our Representation
Without detailed
reaction timings

# Circuit Model of Cytochrome Oxidase C

***Project Leader***

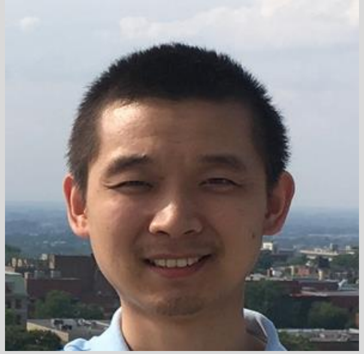
**Shixin Xu**

士鑫 徐



**Huaxiong Huang**

华雄 黄



**Zilong Song**

宋子龙

Xu,  Eisenberg,  Song and Huang (2022). "Mathematical Model for Chemical Reactions in Electrolyte Applied to Cytochrome $ c $ Oxidase: an Electro-osmotic Approach." arXiv preprint arXiv:2207.02215.
Xu,  Eisenberg,  Song and Huang (2023). "Mathematical models for electrochemistry:  Law of mass action revisited " arXiv  2305.12165

## 2 Derivation of Electro-osmotic Model

We mainly focus on a mathematical model of elementary reactions

**Chemical Reaction**

$$\alpha_1 C_1^{z_1} + \alpha_2 C_2^{z_2} + \alpha_3 C_3^{z_3} \underset{k_r}{\overset{k_f}{\rightleftharpoons}} \alpha_4 C_4^{z_4}, \tag{1}$$

where $k_f$ and $k_r$ are two constants for forward and reverse directions, $[C_i]$ is the concentration of $i^{th}$ species, respectively. Here $\alpha_i$ is stoichiometric coefficient, $z_i$ is the valence of $i^{th}$ species and together they satisfy

$$\sum_{i=1}^{3} \alpha_i z_i = \alpha_4 z_4. \tag{2}$$

In particular, we have in mind a case where an active transporter ('pump') uses the energy supplied by a chemical reaction to pump molecules. Later, we will focus on the reaction for cytochrome $c$ oxidase, i.e., for Complex IV of the respiratory chain

**Chemical Reaction**

$$2H^+ + \frac{1}{2}O_2 + 2e^- \underset{k_r}{\overset{k_f}{\rightleftharpoons}} H_2O. \tag{3}$$

According to the conservation laws, we have the following conservation of chemical elements (like sodium, potassium and chloride). Note that this conservation is in addition to the conservation of mass, because nuclear reactions that change one element in another are prohibited in our treatment, as in laboratories and most of life.

**Chemical Reaction**

$$\frac{d}{dt}(\alpha_4[C_1] + \alpha_1[C_4]) = 0, \tag{4a}$$

$$\frac{d}{dt}(\alpha_4[C_2] + \alpha_2[C_4]) = 0, \tag{4b}$$

$$\frac{d}{dt}(\alpha_4[C_3] + \alpha_3[C_4]) = 0. \tag{4c}$$

In order to derive a thermal dynamical consistent model, the Energy Variation Method [89] is used. Based on the laws of conservation of elements and Maxwell equations, we have the following kinematic system

**Field Equations**

**Diffusion, Convection, Migration**

$$\begin{cases} \frac{d[C_1]}{dt} = -\nabla \cdot \boldsymbol{j}_1 - \nabla \cdot \boldsymbol{j}_p - \alpha_1 \mathcal{R}, \\ \frac{d[C_2]}{dt} = -\nabla \cdot \boldsymbol{j}_2 - \alpha_2 \mathcal{R}, \\ \frac{d[C_3]}{dt} = -\nabla \cdot \boldsymbol{j}_3 - \alpha_3 \mathcal{R}, \\ \frac{d[C_4]}{dt} = -\nabla \cdot \boldsymbol{j}_4 + \alpha_4 \mathcal{R}, \\ \nabla \cdot (\boldsymbol{D}) = \sum_{i=1}^{4} z_i [C_i] F, \\ \nabla \times \boldsymbol{E} = \boldsymbol{0}, \end{cases} \tag{5}$$

where $\boldsymbol{j}_l, l = 1, 2, 3, 4$ are the passive fluxes and $\boldsymbol{j}_p$ is the pump flux, $\mathcal{R}$ is reaction rate function. All these variables are unknown and will be derived by using the Energy Variational method.

The total energetic functional is defined as the summation of mix entropy, internal energy and electrical static energy.

**Energy Functional**

$$\begin{aligned} E_{tot} &= E_{ent} + E_{int} + E_{ele} \\ &= \sum_{i=1}^{4} \int_{\Omega} RT \left\{ [C_i] \left( \ln \left( \frac{[C_i]}{c_0} \right) - 1 \right) \right\} dx + \int_{\Omega} \sum_{i=1}^{4} [C_i] U_i dx + \int_{\Omega} \frac{\boldsymbol{D} \cdot \boldsymbol{E}}{2} dx. \end{aligned} \tag{10}$$

Then the chemical potentials could be calculated according to the variation of total energy

$$\tilde{\mu}_l = \frac{\delta E_{tot}}{\delta [C_i]} = RT \ln \frac{[C_i]}{c_0} + U_i + z_l \phi e, l = 1, \cdots, 4. \tag{11}$$

It is assumed in the present work that dissipation of the system energy is due to passive diffusion, chemical reaction and the deduction that energy supplied for pump. Accordingly, the total dissipation functional $\Delta$ is defined as follows

**Dissipation Functional**

$$\Delta = \int_{\Omega} \left\{ \sum_{j=1}^{4} |\boldsymbol{j}_i|^2 + RT\mathcal{R} \ln \left( \frac{\mathcal{R}}{k_r [C_4]^{\alpha_4}} + 1 \right) \right\} dx - \int_{\Omega} f_p dx, \tag{12}$$

where $f_p = f_p(\mathcal{R}, \mu, x) \geq 0$ is the term induced by energy absorption in the pump.

For open systems, especially flux (current) clamp system, in which some fluxes flow in or out, entering or leaving the system altogether, we have the following generalized energy dissipation law

**Dissipation Principle**

$$\frac{dE_{tot}}{dt} = J_{E,\partial\Omega} - \Delta. \tag{13}$$

Here $J_{E,\partial\Omega}$ is the rate of boundary energy absorption or release that measures the energy of flows that enter or leave the system altogether through the boundary. Recall that the chemical potential of a species is the energy that can be absorbed or released due to a change of the number of particles of the given species and $J_i \cdot n$ is the total number of $i^{th}$ particles passing through the boundary, per area per unit time. We define $J_{E,\partial\Omega}$ as follows

# An Electro-osmotic Model of cytochrome c oxidase

The concentrations and potentials at E242, BNC and proton loading site (PLS) and potentials in N and P sides (see Fig.(1) (a)) are modeled using the variables $\phi_E$, $\phi_B$, $\phi_x$, $[H]_E$, $[H]_B$, $[H]_x$, $\rho_e$.

**Field Equations**

$$\frac{d[H]_E}{dt} = \frac{S_v}{F}(I_{N\to E} - I_{E\to X} - I_{E\to B}), \tag{36a}$$

$$\frac{d[H]_B}{dt} = \frac{S_v}{F}(I_{E\to B} + I_{N\to B}) - 2\mathcal{R}, \tag{36b}$$

$$\frac{d[H]_X}{dt} = \frac{S_v}{F}(I_{E\to X} - I_{X\to P}), \tag{36c}$$

$$\frac{d\rho_e}{dt} = \frac{-S_v}{F} I_e - 2\mathcal{R}, \tag{36d}$$

$$C_E\frac{d(\phi_E - \phi_N)}{dt} = (I_{N\to E} - I_{E\to X} - I_{E\to B}), \tag{36e}$$

$$C_B\frac{d(\phi_B - \phi_N)}{dt} = I_{E\to B} + I_{N\to B} + I_e, \tag{36f}$$

$$C_X\frac{d(\phi_X - \phi_P)}{dt} = (I_{E\to X} - I_{X\to P}), \tag{36g}$$

$$C_m\frac{d(\phi_N - \phi_P)}{dt} + I_{leak} + I_{X\to p} + I_e = 0, \tag{36h}$$

with currents

**Structure and Boundary Conditions**

$$I_{N\to E} = max\left(g_D\left(\phi_N - \phi_E - \frac{RT}{F}\ln\frac{[H]_E}{[H]_D}\right), -SW_0\right) = max\left(\frac{g_D}{F}(\mu_N - \mu_E), -SW_0\right), \tag{37a}$$

$$I_{N\to B} = g_K(\phi_N - \phi_B - \frac{RT}{F}\ln\frac{[H]_B}{[H]_N}) = \frac{g_K}{F}(\mu_N - \mu_B), \tag{37b}$$

# More Structure and Boundary Conditions

$$I_{D\to B} = g_B(\phi_E - \phi_B - \frac{RT}{F}\ln\frac{[H]_B}{[H]_E}) = \frac{g_B}{F}(\mu_D - \mu_B), \tag{37c}$$

$$I_{X\to P} = g_X(\phi_X - \phi_P - \frac{RT}{F}\ln\frac{[H]_P}{[H]_X}) = \frac{g_X}{F}(\mu_X - \mu_P), \tag{37d}$$

$$I_{E\to X} = I_{pump} + I_{xleak} = P_{pump}(R_c)(\mu_X - \mu_E) - g_E(\mu_X - \mu_E), \tag{37e}$$

$$I_e = -FJ_e, \tag{37f}$$

$$I_{leak} = g_m(\mu_N - \mu_P) = g_m(\phi_N - \phi_P - E_{other}), \tag{37g}$$

$$I_{E\to X} = I_{pump} + I_{xleak}, \tag{37h}$$

$$I_{xleak} = -g_E(\mu_X - \mu_E), \tag{37i}$$

$$I_{pump} = \begin{cases} g_{pump} max(R_c, 0)(\mu_X - \mu_E), \mu_X - \mu_E < \delta_{th}, \\ g_{pump} max(R_c, 0)\delta_{th}\exp\left(-\frac{(\mu_X - \mu_E)}{\varepsilon}\right), \mu_X - \mu_E \geq \delta_{th}, \end{cases} \tag{37j}$$

$$\mathcal{R} = k_f[H^+]^2[O_2]^{1/2}\rho_e^2 - k_r[H_2O]. \tag{37k}$$

# Preliminary Parameter Values

| Variable | Notations | Values (with Unit) |
|---|---|---|
| $E_{242}$ site effective capacitance | $C_D$ | 1E-1 $fAms/mV/(\mu m)^2$ |
| BNC site effective capacitance | $C_B$ | 1E-1 $fAms/mV/(\mu m)^2$ |
| PLS site effective capacitance | $C_X$ | 1E-1 $fAms/mV/(\mu m)^2$ |
| Membrane capacitance | $C_X$ | 7.5E-2 $fAms/mV/(\mu m)^2$ |
| D channel conductance for $H^+$ | $g_D$ | 3.75E-3$pS/(mum)^2$ |
| K channel conductance for $H^+$ | $g_K$ | 1E-3 $pS/(\mu m)^2$ |
| E2B channel conductance for $H^+$ | $g_B$ | 5E-2 $pS/(\mu m)^2$ |
| E2X channel conductance for $H^+$ | $g_E$ | 1E-3 $pS/(\mu m)^2$ |
| E2X Pump rate for $H^+$ | $g_P$ | 369 $pSms/(\mu m)^2\mu M$ |
| X2P channel conductance for $H^+$ | $g_X$ | 9.8E-4 $pS/(\mu m)^2$ |
| Membrane conductance for leak | $g_m$ | 1$pS/(\mu m)^2$ |
| Mito. matrix $H^+$ concentration | $[H]_{mat}$ | 0.01 $\mu M$ |
| Mito. inner membrane space $H^+$ concentration | $[H]_{ims}$ | 0.063 $\mu M$ |
| Nernst Potential due to other Ions | $E_{Other}$ | $-160mV$ |
| Reaction site $[O_2]$ concentration | $[O_2]$ | 0.0028 $\mu M$ |
| Reaction site $[H_2O]$ concentration | $[H_2O]$ | 0 $\mu M$ |
| Electron current | $I_e$ | -5.24 $fA$ |
| Forward reaction rate coefficient | $k_f$ | 1333 |
| Backward reaction rate coefficient | $k_r$ | 0.005 |
| surface volume ratio | $S_v$ | 1000 |
| Potential Threshold | $\delta_{th}$ | 210 mv |
| Decay rate | $\varepsilon$ | 1 $(ms)^{-1}$ |

Table 2: Parameters

| Variable | Notations | Values (with Unit) |
|---|---|---|
| $E_{242}$ site $H^+$ concentration | $[H]_E$ | 0.01196 $\mu M$ |
| BNC site $H^+$ concentration | $[H]_B$ | 0.01682 $\mu M$ |
| PLS site $H^+$ concentration | $[H]_X$ | 0.01441 $\mu M$ |
| BNC site electric density | $\rho_e$ | 0.01166 $\mu M$ |
| $E_{242}$ site electric potential | $\phi_E$ | -5 $mV$ |
| BNC site electric potential | $\phi_B$ | -14.1562 $mv$ |
| PLS site electric potential | $\phi_X$ | 200 $mv$ |
| N site electric potential | $\phi_N$ | 0 $mv$ |
| P site electric potential | $\phi_P$ | 160 $mv$ |

Table 1: Default Initial Values

# Early Results

# Everything can be Computed.

**Model can be modified to deal with other information and predict experiments**

**Not yet digested!**