Circuits, Magnetism, and Relativity are Inseparable

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Abstract

Circuits outnumber all other applications of electricity. There are more than five billion circuits in a typical smartphone in 2025. Circuits involve current. The laws that govern current flow are Kirchhoff equations and their generalizations derived from the Maxwell Ampere law. The Maxwell Ampere law defines magnetism. The right side of that law is the source of the magnetic field and forms the only intrinsic definition of what Maxwell called "the true current". The true current never accumulates in any time interval. The flux of charge is a component of true current. Flux does accumulate because it reflects the velocity of charge movement. Velocity is a relative measure: if the observer moves along with the charge, at the velocity of charge movement, then velocity, charge movement, and magnetism seem to disappear. Forces do not change as the observer moves at constant velocity: the Lorentz contraction of Einstein relativity is the result. Mass, length, and time then vary with velocity. Charge on an electron does not. Circuits, currents, magnetism, and relativity are inseparable, entwined by Maxwell's equations.

Circuits and currents are inseparable. Through Maxwell's equations, they are entwined with magnetism and relativity, as we shall see.

Circuits make our world.[1, 2] Electrical circuits bring power to our lives, lighting the night. Electron circuits store miles of bookshelves in a few chips.[3] Ionic circuits power our brains.[4-6] Electron circuits speed our thoughts because they are billions of times faster [7-9] than the ionic circuits of our brains.

Circuits outnumber all other applications of electricity. There are nearly ten billion smartphones on earth in 2025 each containing more than five billion circuits. There are then more than 50 $\times$ 10^{18} circuits in our world. Surprisingly, textbooks of electricity fail to mention these facts.[10, 11] Indeed, some prominent popular texts do not even include circuits in their index.[12, 13]

Circuits—whether lighting our lives or illuminating our thoughts—involve current and the laws that govern current flow. Most of our computer circuits are designed [1-3, 7-9] by the simple current laws of the Kirchhoff equations, originally derived one hundred fifty years ago [14-16] for signals in telegraphs. Telegraph signals vary in a tenth of a second at the fastest. Signals today vary in a nanosecond or faster, at least one hundred million times faster [1-3, 7-9] than telegraph signals. Electricity at these speeds behaves differently from electricity in telegraph wires, as is obvious from any measurement: measurements on the nsec time scale always show transients. Measurements on the 0.1 sec time scale do not. Kirchhoff's law in its classical form [14, 17] applies only to the 0.1 sec time scale. It does not allow for transients when used with the Ohm's law definition of a resistance [18-20]. Computer circuits try to remedy this defect with *ad hoc* fixups that describe some transients. It seems that the classical Kirchhoff current law needs to be reworked and derived from the fundamental laws of electrodynamics that include transients, and are true on all time scales, as far as we can measure.[19]

The fundamental laws of electrodynamics are the Maxwell equations [13, 21, 22] that describe electricity without measured errors on any time scale. The Maxwell equations can be written without material constants and involve no adjustable parameters when written that way.[23]

The Maxwell Ampere law defines magnetism as the result of the total current. Total current is what Maxwell identified as “the true current” [24] which according to his Ampere law must be used to understand electrodynamics. Current defined this way is not the flux of charges. Total current includes the flux of charges as one component, along with displacement current of matter $(\varepsilon_r - 1)\varepsilon_0\, \partial\mathbf{E} / \partial t$ and the universal displacement current of space $\varepsilon_0\, \partial\mathbf{E} / \partial t$.[1]

True current never accumulates, not ever, in any time interval or on any time scale [25] that has ever been measured. Flux of charges does accumulate with important consequences. Indeed, in biological systems separate mechanisms made of separate proteins are often needed to deal with the accumulated fluxes. The Na/K ATPase (i.e., the ATP driven sodium pump) needed to recover from action potentials and maintain cell volume is an example.

In a wide variety of models, the flux of charges is the charge density times the velocity of movement of that charge. Fluxes are extrinsic to the Maxwell equations themselves—only current ***not flux*** is a variable in the core Maxwell equations—and so fluxes need to be defined in constitutive equations that are models of material properties ,e.g., p. 220 of [13].

Flux and current, like the equations of magnetism, depend on the definition of velocity. Circuits, current, magnetism, and velocity are thus indivisible. They are inseparably entwined by Maxwell’s equations.

Velocity is defined by the change in distance between two objects. It is the motion of a point relative to an observer. The observer could move at the same velocity as the charges. In that case, relative velocity seems to disappear, current seems to disappear, and so does magnetism [22, 26, 27], as particularly well-explained in Chapter 12 of reference [13].

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[1] ε_r is the dielectric constant, ε_0 is the electrical constant; $\mathbf{E}$ is the electric field.

In his principle of relativity, Einstein postulated that forces are independent of the velocity of an observer. In that case, the Maxwell equations imply the Lorentz transformation and the theory of special relativity.[13, 26, 27] Length, time, and mass vary with velocity according to the theory of relativity, confirmed in experiments of great precision. Charge on an electron does not. See Section 13-6 of reference [22] for an intuitive explanation.

Velocity has a special place for Einstein and his relativity.[28] In his view, currents, magnetism, relativity, and thus—we conclude—circuits are indivisible, entwined by Maxwell's physics and Einstein's God of order and harmony.

But "One man's physics is another man's God," to twist Nikola Tesla's saying. Experiments define physics, in my view, and they do not give velocity the special place Einstein gave it. Experiments (and everyday experience) show that forces ***do*** depend on some kinds of relative motion. Forces depend on the second derivative of position—acceleration. Forces might depend in all sorts of ways on derivatives of motion, some yet to be discovered.

Not for the first time, a beautiful simplification—of circuits, currents, magnetism, and relativity—bows to complexity when examined objectively by measurement.

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