True on All Scales: A Tribute to Stuart Rice

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Abstract

Stuart Rice changed my life when he stood up in the middle of a lecture of mine to say, “The Maxwell equations are true on all scales.” It is rare in science for something to be universal, true for things both atomic and stellar, so I did not understand his words at first. After some 30 of years of study, I now know Stuart was prescient: the Maxwell equations are universal because they embody the theory of relativity. One of the Maxwell equations ***is*** precisely the same on all scales. The Maxwell Ampere equation creates a solenoidal current field without sources or sinks. It thereby creates the circuits of our kitchen, the circuits in nerve cells, and the circuits linking continents in cables under the ocean. Circuits power life in the electron transport chains of our mitochondria. Circuits allow our computers to work on the atomic scale, essentially without error, at little cost. Thank you Stuart for seeing and pointing the way.

Sometime around 1995 Stuart Rice of the University of Chicago invited me to give a seminar in Chemistry about ion channels. It started at noon, on a Friday, I think. Most of what the chemists knew about molecular biology was qualitative. My task was to convince them that the biophysical study of ion channels provided reproducible data as accurate and interpretable as in physical sciences. And, of course, I hoped to persuade the chemists that our physical approach to electrical models of the open channel was acceptable physical chemistry. After all, it was based on well-established models of flow of current in PNP transistors, using a physical model for ion diffusion we called PNP (**P**oisson **N**ernst **P**lanck)[1].

When Stuart brought me to the seminar room, I was surprised to see only a handful of people: Graham Fleming and Steve Berry and postdocs.

The seminar was great fun. There were constant interruptions as the audience asked for more about quantitative channel biophysics as set forth by the Nobelists Alan Hodgkin, Andrew Huxley, and Bernard Katz. Fortunately, I could answer the questions. I had spent a lifetime in the lab working in that tradition. I started studying channels in 1959 as a first-year undergraduate at Harvard College, in John Pappenheimer's graduate course on Hodgkin Huxley at Harvard Medical School, which I later found out was supposed to off-limits for undergraduates!

The seminar's intellectual focus was on the electrical properties of ion channels, the electrical current that flows through them, and how to explain that with classical engineering circuit theory. Electrical phenomena were only a peripheral interest for Stuart, Graham, and Steve. To understand circuit theory or ion channels, they also needed the idea of a device, with inputs, outputs, and mathematically defined input-output relation.

The idea of a device is central to our technology (and thus our economy) but is surprisingly unknown to most physical scientists. Biophysicists grew up with the idea because ion channels are devices and biophysicists of my generation studied their input-output relations every day.

Before I knew it, my scheduled 50 minutes were up. I started to wind up around, but Stuart said, “Please go on.” I asked, “Are you sure?” All said yes … and the questioning continued. I tried to explain how electric circuits worked in nerve cells that could be a meter long or 10 μm in diameter and even in individual protein molecules (ion channels) 2 nm long. My collaborators and I applied the same approach even on the atomic scale. Physical chemists study electricity on the atomic scale without circuits (and mostly at equilibrium) and so I was afraid our circuit approach would be ridiculed when used on the atoms of ion channels. Stuart encouraged me to tell them how we did it.

I went on and on, having a great time. The audience was enjoying it too, judging from how they peppered me with hard questions, particularly Stuart and Steve. Then, lecturing suddenly paused after an hour and forty-five minutes. Stuart stood up while I was in midsentence and said something like, “I have to meet with the provost but please keep going with your talk.” He said, “Good talk, but you do not have to worry so much about using circuit laws to describe your ion channels.” He then said, **“Maxwell’s Equations are true on all scales.”** No one knew what he meant, certainly not me.

The audience insisted I continue without Stuart. Their questioning continued vigorously, to my delight. The seminar ended half an hour later, when I suspect physiological hunger overwhelmed our intellectual appetites.

Some weeks later Stuart told me what he meant. He explained that some electrodynamics were universal. Thus, some version of Maxwell’s equations was true on all scales, although the right approximation and formulation might vary from condition to condition. As Stuart put it, in one of his characteristic comments, the approximations make a theory chemistry, not mathematics.

My approximations must have pleased him. Stuart and Steve invited me to add some pages [2] to the second edition of their textbook of Physical Chemistry. And Stuart invited me to write a long review for Advances in Physical Chemistry [3] that introduced the idea that crowding was the key to understanding ions in channels, proteins, and batteries.

Over the next thirty-four years, as I came to understand Stuart's words "Maxwell's equations are true on all scales", my life was changed [4-27]. I found that one part of those equations was not only true, but it was exactly the same on all scales from atoms to stars. "True on all scales" was a mathematical property of one of Maxwell's equations, called the Maxwell-Ampere law and its corollary that I call the Maxwell current equation. The current equation did not need a chemical or physical approximation to be true on both the stellar and atomic scale. Current satisfied a solenoidal field equation, in mathematical language, that was independent of the properties of matter and so it was true on all scales.

My life was changed by this effort as I will try to explain briefly without being very technical. More importantly, it has led to a new understanding of everything electrical, including the computers so important to our lives.

I had decades of ignorance and then confusion until I understood what "Maxwell's Equations are true on all scales" actually means [4-22]. The eventual answer [23-27] turns out to be exact in important aspects, not depending on approximations at all [27]. It includes both interstellar (and perhaps intergalactic distances) and atomic distances, where quantum mechanics is involved. The quantum mechanics is novel and was chiefly the work of my collaborators David Ferry and Xavier Oriols, with input and help [7, 21, 28] [Dave and Xavier are nearly as famous in the quantum world as Stuart was in physical chemistry so the result has a secure pedigree extend beyond my expertise and experience.]

My first task was to learn about the Maxwell equations that Stuart was talking about [6, 7]. This was harder than it seemed. The Maxwell equations in the textbooks were clearly not universal or true on all, because those classical equations involved approximations. They are part of what Stuart would call "chemistry not mathematics". The classical Maxwell equations are constitutive equations, as mathematicians would call them, that describe properties of matter with approximations that are adequate only in a range of conditions. The classical equations are obviously untrue over all scales–particularly in the ionic solutions of physical chemistry and physiology–because they do not begin to fit huge amounts of experimental data, even qualitatively [9, 29]. The classical equations try to describe all that data with one dielectric constant, whereas experiments since the 1920s showed that the ‘constant’ varied dramatically with the contents and concentrations of the ionic solutions, as well as with time and lots else.

It was not at all clear how to best write a version of the Maxwell equations that was true on all scales [8, 10, 11, 13-16, 18, 19]. Indeed, the best way to write them is still not known [19] although several versions are good enough so we can proceed [23, 26, 27, 30, 31].

It took many years to learn to write the equations so I could begin to consider them on all scales. The problem was that Maxwell combined the properties of matter–technically called the polarization of dielectrics–with the universal properties of electricity. The properties of dielectrics are not true on all scales, even qualitatively [9, 29]. The properties of dielectrics include all the ways matter can move when electric forces are applied. These are far too diverse to describe with a constant and in fact probably cannot be described by any single theory [28]. Some of the properties of the Maxwell equations depended on these properties. Some did not. The problem was how to separate them, how to determine which was which.

This is a familiar challenge in science. Scientific results depend on models like that of a dielectric that are only approximately true within certain conditions. Hardly a surprise! Results and models change as you move from stars to animals to cells to proteins to atoms. It is almost always necessary to have approximate models. If you try to remove all approximations, the model usually vanishes. That is to say, without the approximations you cannot make predictions about the real world that can be compared to experiments. Without approximations, science usually cannot progress very far. It can produce almost nothing useful for technology, for example.

The Maxwell equations turned out to be different, utterly different [16]! Part of the Maxwell equations, called the Maxwell Ampere law, tells you that circuits exist—like those in our computers–on all scales, without approximation. The Maxwell Current Law is a result of mathematics not physics. Yes, some parts of the Maxwell equations depend on scale and approximation just like other models (or perhaps I should say that I do not know how to remove the scales and approximations). Amazingly, though, **a crucial part of the Maxwell equations is entirely independent of scale and properties of matter,** and I could prove that by mathematics, without approximation! [Technical note: the Gauss' law in the Maxwell equations depends on scale and properties of material charge; the Maxwell Ampere law and the Maxwell Current Law do not. Circuits depend on the Maxwell current law, not Gauss' law. That is why circuits exist on all scales, centimeters long in our kitchens, nanometers in our computers, a handful of atoms in our ion channels, and thousands of kilometers in telegraphs [32] and trans-Atlantic cables [33, 34] that form 'a thread across the ocean' [35], to make a 'Victorian Internet' [32].]

Stuart was surprised and skeptical of this result, as he should have been. It is only fair to say that he was not confident of my (or perhaps his own) facility with the vector calculus of fields to be sure the mathematics was flawless. I was skeptical as well [16] and went to mathematicians to check the results [7, 17, 19, 21, 28]. Some of the mathematicians were as familiar with vector calculus [8, 31, 36-42], as Stuart was with physical chemistry.

Stuart knew that electrical phenomena occurred on all scales, and he knew that one should make models on all scales. What he and I did not imagine possible was that the Maxwell Ampere Law, and what I came to call the Maxwell Current Law [27], would be true on all the scales from stars to inside atoms **without any change at all.** What Stuart had stated as a chemical approximation in 1995 was prescient. "True on all scales" turned out to be a theory of everything about electrical circuits. It was a TOE that could be derived from the Maxwell Ampere law with no approximation.

The Maxwell equations do not behave the way other models I had studied since 1959. It was possible to write a crucial part of the Maxwell equations in a way that did not depend on the properties of matter at all!! That part of the Maxwell equations was true on all scales and in only one form! There were no material parameters in the Maxwell Ampere law, and what I came to call the Maxwell Current Law [27]. The only parameters were the speed of light and the electrical or magnetic constant. All these were known to be constant to fantastic accuracy under all conditions that have been measured. These parameters appear because they appear in Einstein’s theory of relativity [43-45] when you consider the relative motion of an observer and the stream of moving charges that form an electric current. That system is calculated in detail in textbooks of electrodynamics [46-49].

The Maxwell Ampere equation is the name of the crucial part of the Maxwell equations true on all scales. The Maxwell Ampere equation depends on no properties of matter at all. **And it still had useful content important for practical life because it has a very special mathematical property, because it describes what is called a solenoidal field**, in which current flows without source or sink, in endless loops as it circuits through material or empty space**.** In fact, the computers we all use work because of those endless circuit loops.

Maxwell himself was quite aware of the special properties of current produced by the Maxwell Ampere Law. In uncharacteristic judgmental language, Maxwell gave the total current the special name ‘true current’. He used the judgmental adjective ‘true’ in this name. It is the only moral adjective I could find in the 868 pages of his Treatise on electrodynamics [50, 51]. Indeed, Maxwell explicitly instructed his readers in Vol. 2, Section 610 p. 232, of [50] that to understand electricity they had to understand true current. He then used many pages to explain his instruction, giving practical examples of what went wrong if you did not define true current as he demanded. See reference [27] for an explanation in modern language and mathematical notation. Maxwell’s statement was unknown to both Stuart and me (I suspect) because it was not properly indexed in the standard editions of the Treatise. I had to generate a searchable PDF file of [50] to find it. The many pages of Section 610 are prominent but only after you find them!

The special solenoidal nature of current flow that Maxwell identified so long ago is not an abstract part of science. **It is exactly the part of Maxwell’s equations that describes the rapid transient currents in the circuits of our computers** that change in 0.1 nsec or less**.** Electrical charges (think electrons) flow through our devices in endless loops called circuits. The charges flow on all scales no matter what the circuit is made of, no matter how large or small the circuit is, or how fast or slow is the flow. That is a fundamental property of the Maxwell equations, true on all scales. Amazingly, Stuart’s statement “Maxwell’s equations are true on all scales” was exact! And eventually I could prove that using mathematics alone, without approximation.

When Stuart told me, “Maxwell is true on all scales,” I believe he meant to motivate me to keep using the circuit approach **to look for the approximations needed** to make models work even in my ion channels. So, I believe he knew the approximations would be hard or impossible to find in the classical physical chemistry of equilibria and he wanted to encourage me to look elsewhere. **Neither of us expected that we would find a universal statement, a ‘TOEec’ theory** (of everything about electrical circuits) **that was exact on all scales, as exact as any theory in science.** I named the TOEec ‘Maxwell’s Current Law’ and hope it will come to be used widely as an exact generalization of the well-known widely used (and very approximate) Kirchhoff current law [8, 10, 23, 26-28].

This TOEec is important practically. It matters. The circuits in our computers switch every 10^{-10} seconds or faster. They switch 3.6×10^{12} times in an hour. There are more than 10^{9} circuits in our cell phones. **A single error in one computer switch will often crash the entire system.** Computers could not be used if they crashed every hour. Mathematics, science, and engineering work together to prevent those errors.

Mathematics forces current to flow in endless solenoidal loops. It cannot flow any other way: it has no sources or sinks. The science of physics shows how to isolate circuits, so they barely interact. That allows the science of engineering to control the circuits independently, one by one, making the logic elements of our computers, for example. These sciences work together to allow our computers to work on the atomic scale, essentially without error, at little cost.

Computers are possible only because they do what they are supposed to. Computer circuits follow the Maxwell equations. Stuart was more right than he knew at first.

And I am forever grateful he showed me the question to ask and encouraged me for the many years it took me to show he was right, helped along the way by other fine scientists as well.

In Memoriam

The importance of Stuart's comment 30 years ago became clear to me at the memorial service following his passing from us. Only then did I realize that he must have guided me with purpose, helping my intellectual ambition to flow as it did, even though that flow led away from much of his life's work.

My relation with Stuart was complex for me because I respected him too much to say what I thought were the limitations [18, 52] of his approach to physical chemistry, highlighted in the title of the first edition of his textbook "Physical Chemistry: Matter in Equilibrium" [53]. In my view, the equilibrium approach had to be replaced something more realistic that involved flow. Flow is forbidden in equilibrium analysis.

Modern life is possible because of engineering devices that always involve flow. But equilibrium assumes zero flow. Once flow is assumed to be zero, it is hard to make it be anything else. The equilibrium approach had to be replaced if physical chemistry is to remain a major part of contemporary science. Stuart's life work on equilibrium systems cannot be extended in general to the nonequilibrium technology that makes modern life possible.

During our friendship I had never said the words "The equilibrium approach must be replaced." But the papers he solicited [2, 3] from me said that. He read, criticized, and refereed them thoroughly. But I had never said those words aloud to him. I had too respect for Stuart to do that.

It was only after his death, at his memorial service, that I realized what Stuart must have recognized all along. He had encouraged me to follow the path of flow (not equilibrium) for more than thirty years, from our first to last conversation, even though he knew it would upset the equilibrium of his life's work. He truly fulfilled his wish to be remembered as a good man, both intellectually and morally.

I am glad Stuart vigorously interrupted my lecture so many years ago, and I listened.

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