Molecular Vortices Abandoned:

Polarization, Maxwell, and His Equations

Robert S. Eisenberg

Bob.Eisenberg@gmail.com

February 1, 2026

DOI: 10.13140/RG.2.2.15017.45922

Department of Applied Mathematics

Illinois Institute of Technology

Chicago IL USA

Department of Physiology and Biophysics

Rush University Medical Center

Chicago IL USA

Abstract

When Maxwell abandoned his theory of vortices, he replaced molecular models with an abstract mathematical model of the polarization field as $\mathbf{P} = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$. The polarization field has been used throughout electrodynamics since then. This model of polarization was not entirely abstract: it used a single 'dielectric constant' to describe a vacuum, just ε_0 by itself. It used a different dielectric constant ε_r, a positive real number $\varepsilon_r \geq 1$, to describe charge in matter. But a single dielectric constant does not have enough resolution to describe the enormous range of motions of charge that can occur when electric fields are changed. Indeed, $\mathbf{P}$ does not even specify a high-resolution model uniquely for a given structure of matter. A $\mathbf{curl}$ function can always be added to it with no effect on the electrical forces that depend on $\mathbf{div}\,\mathbf{P}$, e.g., in the Gauss partial differential equation. Polarization phenomena need to be described more generally, the way compressibility of mass is described in fluid dynamics. A constitutive phenomenological model is needed. Most properties of fields depend on the details of constitutive phenomenological models. Amazingly enough, the Maxwell-Ampere law and its current law do not. They specify the total (true) current defined by Maxwell. The total true current is a solenoidal field divergence-free on an extraordinarily wide range of time scales, including those of electron orbital changes in chemical reactions. Our electronic technology exploits the special properties of the streamlines of the solenoidal fields of total current. The true current in finite two-dimensional systems—like circuits—flows in loops and returns to where it started under all conditions and at all times. These nearly universal properties help ensure the breathtaking success of practical electronic circuits over some ten orders of magnitude of time. Chemical reactions often occur in groups, producing currents that flow in circuits according to circuit laws, but theories of chemical reactions usually ignore electrodynamic constraints on total current.

Electrodynamics and polarization are inseparable. Electric fields polarize insulators by changing the distribution of charge. Polarization changes the electric field. Electric field changes polarization. At first glance, it might seem that the application of electrodynamics requires knowledge of polarization, but we will show that is not the case for circuits. Circuits are governed by current laws eq.(3)&(4) that do not involve material polarization. The current laws only involve the polarization of space, e.g., a vacuum. The Kirchhoff current law eq. (4) was used on the 10^{-1} second time scale of the telegraphs of the 1840s. The same current law eq. (4) is used today on a 10^{-10} second time scale in our computers and phones.

Circuits. Although telegraph circuits were the most important application of electricity in their time, Faraday and Maxwell were mostly concerned with fields, not circuits. The historical literature can be reached through Siegel [1] (Ch 4 and 6), Bromberg [2], and Rosenfeld [3]. Circuits are neglected in electrodynamics even today despite the billions in each of our phones and computers. The word "circuit" is not found in the index of the leading textbooks of electrodynamics, whether introductory [4] or comprehensive [5].

Maxwell and Faraday had a reason for not emphasizing circuits. They had an imperfect understanding of the current that flows in circuits. They did not recognize the existence of particles with a constant electric charge—like electrons, protons, quarks, or ions—for reasons that are hard to understand today. Today, electrodynamics is usually taught as the properties of particles with constant charge, electrons in wires, and ions in water, for example. Ionic solutions are characterized by the concentration of charges. Concentration is defined by the number of moles (of constant charges) in a volume. A mole of charges is "the Faraday". The Faraday is the universally used unit [6] defined by the total charge of 6.023×10^{23} (Avogadro's number) of elementary charges. The irony is that Faraday himself did not believe that elementary charges existed, and Maxwell rejected the possibility of their existence in three pages of his Treatise [7] (Sections 261 and 262, p.311-313), using the tone of ridicule more than analysis.

Maxwell says that "constant molecular charge is a gross phrase, out of harmony with the rest of this treatise" [7] (Sections 261). Admirably honest about the limits of knowledge

concerning the 'true nature' of electricity, Maxwell nonetheless prophesizes, incorrectly, "It is extremely improbable that when we come to understand the true nature of electrolysis we shall retain in any form the theory of molecular charges". Today, however, we know that the true nature of ionic solutions (and electrolysis) depends on the charges of their individual molecules, which are usually constant [6, 8-10]. No wonder Maxwell had difficulty dealing with charge or current!

Molecular Vortices. Maxwell started his study of polarization with a theory of matter using molecular vortices without definite charge.[1] He struggled with his theory of matter in a series of four papers published over thirteen months, between February, 1861, and March, 1862. See [1], and p. 185 footnote 1, for documentation and admirable discussion of the four Maxwell references. All are available in the compilation [7]. Once he abandoned molecular vortices, Maxwell's field theory became his partial differential equations [11], as in the Treatise [7].

Amazingly, the equations Maxwell wrote—without understanding or using the existence of ions or electrons—are the electrodynamics still used today in a world where particles and subatomic particles have definite charge, whether ions, electrons, or quarks. One is tempted to reiterate the obvious: Maxwell's path to discovery was quite different from our path to understanding some 160 years later. According to Siegel [1], Maxwell was driven by the need for mathematical precision, and thus consistency, even more than by his vision of molecular vortices.

The mathematically precise theory Maxwell created survived his ignorance of charges of fixed size but the struggle with molecular vortices left important residues. The polarization field $\mathbf{P} = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$ is one residue. It has survived the abandonment of vortices.

The polarization field $\mathbf{P}$ defined by Maxwell has been used ever since to introduce students to electrodynamics of charge in matter. That polarization field $\mathbf{P}$ is widely used as an initial oversimplified model of new systems. As those models are refined by measurements over more

[1]Maxwell believed that true current was the fundamental measure of the total movement of electricity, and that true current ***always*** included a term that depended on the time derivative of the electric field—called the displacement current. Thus, a single particle of electricity could not have a single definite value independent of the time derivative $\partial\mathbf{E}/\partial$t. See Discussion Section.

conditions, $\mathbf{P} = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$ is replaced with more complex and realistic descriptions of the polarization phenomena [12, 13].

Single dielectric constant. The polarization field $\mathbf{P}$ used a single dielectric constant ε_r for each type of matter . But that approximation does not have enough flexibility or resolution to describe the enormous range of motions of charge that occur when electric fields are changed. Reference [14] discusses these issues and is an introduction to the huge literature measuring polarization. References [12, 15-24] document the many types of polarization and report their properties. Robinson [13] describes the classical treatment of complex time dependence in linear systems. Parsegian [15] p. 241-275 shows how optical properties are an expression of polarization. The optical properties and polarization of almost every organic chemical are different. Google reports that 20 million organic chemicals are known which implies 20 million different spectra of charge polarization, each requiring a different dielectric parameter $\varepsilon_r(\omega)$ at each frequency ω of the spectra of each chemical.

A general analysis of polarization phenomena does not seem available, or possible, but a wide-ranging energetic treatment [25] comes from the Lagrange multiplier treatment of the energetics of polarization. It includes the phenomenon of drift diffusion used to design the transistors and integrated circuits of our technology [26-39], and the nearly identical PNP (Poisson Nernst Planck) treatment of ionic solutions [40-43] in bulk and proteins of biology.

P is ambiguous. A general analysis seems impossible because the very idea of a unique $\mathbf{P}$ has inherent difficulties on a molecular scale.

As Purcell and Morin discuss in detail (p. 500-507) [44], many different atomic structures can give the same $\mathbf{P} = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$: "Can one always identify unambiguously the "molecular dipole moments" in matter, especially solid matter? The answer is no."

Purcell and Morin recommend abandoning the classical approach to $\mathbf{P}$. "This example teaches us that in the real atomic world the distinction between bound charge and free charge is more or less arbitrary, and so, therefore, is the concept of polarization density $\mathbf{P}$." They imply that an explicit model is needed for each system of interest.

The ambiguities in the physical treatment of $\mathbf{P}$ come also from the underlying mathematics, as described in detail in [45], eq. 9 and 10. A **curl** function can always be added to $\mathbf{P}$ with no effect on the electrical forces that depend on $\mathbf{div\ P}$ —for example, in Gauss' law—because $\mathbf{div\ curl}$ is always zero [46-49]. An infinite number of structures have the same $\mathbf{div\ P}$ and thus the same distribution of electrical forces. See [50].

Maxwell could not resolve $\mathbf{P}$ with his molecular vortices. Evidently, modern physics cannot uniquely resolve them either. The question is then "Can we do without $\mathbf{P}$? Can we do away with $\mathbf{P}$ and and still say something useful?" It turns out we do not need "exhibit in every case all the charges, whatever their origin" as is widely believed, and implied, more or less, by Feynman [51], Section 13-4.

We can analyze circuits without knowing all the charges because Maxwell's struggle with molecular vortices left another residue, namely the idea of displacement current. Maxwell recognized the importance of displacement current when he identified its discovery as the chief contribution of his Treatise [7]: p. 232 Vol 2.

Displacement current accounts for the propagation of light through a vacuum [4, 5, 44, 51-55]. The displacement current allows calculation of the velocity of light from measurements of the electrical and magnetic constants, which seem to have nothing to do with light. Measurements of electrical and magnetic constants are made in a laboratory. Measurements of light are made in a vacuum. Maxwell himself called displacement current "the chief peculiarity" of his Treatise [7], Section 610, p. 232 Volume 2, perhaps for this reason [1].

As Maxwell struggled with molecular vortices, he made them elastic. Elastic vortices could define a mathematically consistent theory of electrodynamics. Rigid vortices could not. Maxwell found [1] (p. 107, 118) that elasticity in fact implied the displacement current found in Maxwell's extension of Ampere's law. (Possible alternatives to the displacement current are mentioned in Panofsky, [53], p.139.) The Maxwell-Ampere law, along with the continuity equation, allowed a consistent definition of true current.

The continuity equation itself when applied to discrete charges allows the consistent definition of current without consideration of wave propagation in the vacuum. Kirchhoff [56] established this fact, even before Maxwell, as discussed in [57].

Kirchhoff's approach allows a consistent unambiguous treatment of current even in the quantum world [57], despite the ambiguities of measurements in some interpretations of quantum mechanics [58]. The definition of current in quantum devices is of practical importance now that the circuits of our computers involve quantum effects [26-39, 59-61].

The Maxwell-Ampere law eq.(1) is the fundamental law that defines the magnetic field arising from its source, the right-hand side of eq.(1), $\mu_0(\mathbf{J} + \varepsilon_0\, \partial\mathbf{E}/\partial \mathrm{t})$. Remember $\mathbf{div\ B} = 0$.

Eq.(1) implies the existence of a 'true' current that is universal and solenoidal [53], divergence-free at all times and under all conditions, as we shall see. The Maxwell-Ampere law (1) is true for gamma rays (10^{-22} seconds) and probably much faster. Errors in the Maxwell-Ampere equation dealing with photon-photon interactions are probably not of great interest in the present context [62, 63].

Maxwell-Ampere law: $$\mathbf{curl\ B} = \mathbf{J}_{\boldsymbol{true}} = \mu_0(\mathbf{J} + \varepsilon_0\, \partial\mathbf{E}/\partial \mathrm{t}) \qquad (1)$$

The Maxwell-Ampere law involves the magnetic constant μ_0**,** the electrical constant ε_0, the electric field **E,** the magnetic field **B** and the conduction current **J**. The conduction current is defined by the movement of charge with mass, however fast, transient, or brief. The conduction current **J** includes properties of dielectrics implicitly. It includes the material displacement current associated with the usual approximation to material polarization $\mathbf{P} = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$.

Maxwell defined total current as the true current ([7],p.232, Section 610 of Vol. 2).

$$\mathbf{J}_{\boldsymbol{true}} = \mu_0(\mathbf{J} + \varepsilon_0\, \partial\mathbf{E}/\partial \mathrm{t}) \qquad (2)$$

The conduction current $\mu_0\mathbf{J}$ does not include the polarization current of space $\mu_0\varepsilon_0\, \partial\mathbf{E}/\partial t$. The polarization of space that might be called 'ethereal' because it is present in a vacuum and does not arise from matter or charge.

The identity of the carrier of the ethereal current $\mu_0\varepsilon_0\,\partial\mathbf{E}/\partial\mathrm{t}$ is not known to me. The flow of the ethereal current is known, and how much flows is known, but not what flows. The carrier of ethereal current has a name, however. It is the "aether"[64] or "ether"[54] of 19th century electrodynamics that has fallen out of favor since the Michelson-Morley experiment [65]. Modern literature [66-68] discusses the properties of the aether needed to deal with the Michelson-Morley experiment and special relativity.

The Maxwell-Ampere law (1) is profoundly important because it shows that electric fields in a vacuum—devoid of charge or matter and thus with $\mathbf{J} = 0$—produce magnetic fields $\mathbf{B}$. Remember $\mathbf{div}\,\mathbf{B} = \mathbf{0}$. This magnetic field combined with the electric field produces propagating radiation like the light from sun to earth that shines through a nearly perfect vacuum. Details in [4, 5, 13, 44, 51-53, 69-71].

If we take the divergence of eq. (1), we see that the true current $\mathbf{J}_{\boldsymbol{true}} = \mu_0(\mathbf{J} + \varepsilon_r\varepsilon_0\,\partial\mathbf{E}/\partial\mathrm{t})$ forms a divergence-free solenoidal field because $\mathbf{div\ curl}$ is always zero, easily verified by direct substitution in the Cartesian definition of the vector functions and extensively discussed in the mathematics literature from many points of view [46-49, 72].

The Maxwell current law eq. (3) is a mathematical corollary [73] of the Maxwell-Ampere equation (1). It was derived without approximation, without adding physics. It is true whenever the Maxwell field equations are true.

Maxwell Current Law:
$$\mathbf{div}\ \mu_0(\mathbf{J} + \varepsilon_0\,\partial\mathbf{E}/\partial\mathrm{t}) = \mathbf{div\ curl}\,\mathbf{B} = 0 = \mathbf{div}\,\mathbf{J}_{\boldsymbol{true}} = 0 \tag{3}$$

Remember that the conduction current $\mathbf{J}$ includes the material displacement current associated with the material polarization $\mathbf{P} = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$.

The Kirchhoff current law is the low frequency, long-time approximation to the Maxwell current law [73]. It is widely used in conjunction with *ad hoc* stray (parasitic) capacitances that modify topological circuits [30-39] so they can approximate reality including the displacement current term $\varepsilon_r\varepsilon_0\,\partial\mathbf{E}/\partial\mathrm{t}$.

The Kirchhoff Current Law is valid only when $\varepsilon_0\,\partial\mathbf{E}/\partial\mathrm{t} \ll \mathbf{J}$.

$$\mathbf{div}\ \mu_0\mathbf{J}\Big|_{\varepsilon_0\partial\mathbf{E}/\partial\mathrm{t}\ll\mathbf{J}} = \mathbf{div}\,\mathbf{curl}\ \mathbf{B}\Big|_{\varepsilon_0\partial\mathbf{E}/\partial\mathrm{t}\ll\mathbf{J}} = 0 \tag{4}$$

Solenoidal fields of true current. The true current forms a divergence-free solenoidal field according to eq. (3). Solenoidal fields of true current have the special property that the true current never accumulates on any time scale for which the Maxwell-Ampere equation (1) is true. True current cannot be stored to be 'paid back' at a later time. If true current flows in an unbranched one-dimensional system, like a series circuit, it is equal in every component of that series circuit ***at any time*** because it is a solenoidal divergence-free field [74]. In such series systems, there is no need for a spatial variable to keep track of location when studying the true current $\mathbf{J}_{true}$ because there is no spatial dependence of the true current at any time, even on the time scale of thermal motion. The conduction current $\mathbf{J}$ is another matter. It varies with spatial location even in a series circuit, often dramatically.

Circuits. Branched one-dimensional networks form the circuits of our electrical and electronic technologies [26-39, 55, 59-61]. These circuits function essentially without errors at time scales ranging from 0.1 nsec to many hours, in systems of enormous complexity. The systems are finite two-dimensional networks of one-dimensional devices like resistors, capacitors, and transistors.

Streamlines in solenoidal fields. Current flow has special properties because true current is solenoidal, divergence-free. Streamlines of current in solenoidal divergence-free fields have very special properties [48, 49], particularly in finite two-dimensional systems. These streamlines do not have sources in the usual monopolar sense of the word, as in permanent charge sources in Gauss's law. Solenoidal fields and flowlines are very different from the lines of force of classical electrostatic fields driven by a few permanent charges.

The true current flows in streamlines that end where they begin, forming generalized 'circles' called circuits. These idealized topological circuits are used to design our computers and

technology [26-39, 59-61]. Their special mathematical properties are likely to have something to do with their technological success.

The idealized topological circuits are designed with the current laws eq.(3)&(4). Kirchhoff's law eq. (4), and its generalization, the Maxwell current law eq. (3), are not constitutive or phenomenological. They—like the Maxwell-Ampere law from which they are derived—are true for any properties of any material.

Once materially specific properties are banished from the Maxwell equations, and the polarization approximation $\mathbf{P} = (\varepsilon_r - 1)\varepsilon_0\mathbf{E}$ is abandoned, it would be natural to assume that nothing materially specific can be stated, beyond conservation laws themselves. Most scientists assume that electrodynamic models need to "exhibit in every case all the charges, whatever their origin" Feynman [51], Section 13-4. They wonder **if material parameters are absent from the equations (1)**&**(3), how can properties of material circuits be predicted?**

The fact (and wonder) is that circuits are analyzed and built without knowledge of material parameters [26-39, 59-61]. The Maxwell-Ampere law implies the Maxwell current law without any additional physics or mathematical approximation. The Maxwell current law, like the Maxwell-Ampere law depends only on the universal properties of space, including a vacuum. The Maxwell current law eq.(3) is a nearly universal generalization of Kirchhoff's current law.

Surprisingly, we see that the general properties of current in practical electronic circuits [26-39, 59-61] can be described in a universal way, using only field equations that depend on the polarization of free space and are true for any properties of any materials.

Discussion

Molecular vortices abandoned. Once Maxwell abandoned his molecular vortices, he concluded that " … the time-variation of the electric displacement must be taken into account in estimating the total movement of electricity". He could hardly have used more explicit language. He said a time-varying component had to be used when estimating the total movement of electricity. A particle that moves 'total … electricity' must therefore be time-varying, not constant.

We can then see why Maxwell rejected the idea of particles moving with a constant amount of 'total electricity'. For Maxwell, ***the 'total movement of electricity' MUST always include a displacement current and that must vary with time*** because it depends on $\partial\mathbf{E}/\partial$t. The displacement current could not be a constant because $\partial\mathbf{E}/\partial$t was not a constant.

If a particle of electricity existed, Maxwell presumably thought the particle must be a part of the total true current $\mathbf{J}_{\boldsymbol{true}} = \mu_0(\mathbf{J} + \varepsilon_0\,\partial\mathbf{E}/\partial\mathrm{t})$. The particular current could not be constant because ***every part*** of 'the total movement of electricity' must include a displacement component and vary with $\partial\mathbf{E}/\partial$t. Anything else would be " … a gross phrase, out of harmony with the rest of this treatise" [7], Section 610, p. 232 of volume 2.

Chemical reactions. The flow of a single electron from one orbital to another is the essence of a chemical reaction. It occurs in $10^{-15} - 10^{-18}$ seconds. Chemical reactions are often found in groups and involve the flow of currents in circuits that obey the universal laws of current flow, eq. (1) & (3). For example, the flow of total current in two chemical reactions connected in series must be equal at all times [74]. Total current cannot be stored. Circuit properties of this type are not usually included in theories of chemical reactions.

It is fascinating to wonder what would happen if quantum chemistry used a particle of true current as its fundamental representation of a reactant or product. That particle was given the fanciful name 'Max-ion' [75], p.2 in honor of Maxwell. Note the Max-ion moves a variable amount of current because the total current depends on $\partial\mathbf{E}/\partial$t.

Or perhaps a representation of the fundamental particle should also include a permanent magnetic dipole—the ever-present spin of the electron, usually ignored in treatments of chemical reactions. This provides a more complex elementary 'quantity of electricity', that moves as a magnetic ion, a 'Mag-tron'. The Mag-tron combines conduction charge, its displacement current, and permanent magnetic dipole $\pm h/2$ with its singular special properties, p. 3 of [75]. Mag-trons are likely to associate as magnetic dipoles do on macroscopic scales, head to tail, perhaps producing the Pauli exclusion principle when placed in the 'electric' field of the nucleus, e.g., the proton of the hydrogen atom.

Rate constant models of chemical reactions are not transferrable in most cases. They need different parameters—different rate constants—under different conditions, so the models fit data under different conditions. It is rare to be able to predict the value of those rate constants ahead of the experiment. The use of non-transferrable models has been canonized in the chemical literature. The non-transferrable property of chemical models makes chemical devices hard to design [76].

This non-transferrable property of theories of chemical reactions is significantly different from theories of electrical circuits. Electrical circuits fit data over a wide range of conditions with a single set of parameters, transferrable from one setup to another [26-39, 59-61, 73].

Our computers could hardly function if the properties of each transistor varied in an unpredictable way. In my view, non-transferrable models are so difficult to disprove [77] that they should not be considered part of science. Rather, non-transferrable models should be seen as part of the social and psychological culture [78] that surrounds and motivates scientists to do their work.

In my view, the science of chemical reactions must deal explicitly with the displacement currents $\varepsilon_0 \, \partial \mathbf{E}/\partial t$. I suspect that " … the time-variation of the electric displacement must be taken into account in estimating the total movement of electricity" [7] even on time scales 10^{22} times faster than Maxwell considered when he wrote these words. Without the displacement

current, it is hard to see how models of chemical reactions can be as useful as models of electrical circuits have been.

Universal Current Law. Few physical laws are universal. The Maxwell current laws are. They give us a powerful tool for understanding complex systems because they are always true no matter the complexity of the system or the experimental setup used to study the system. When total current is the output of a system, the Maxwell current laws allow complete analysis of systems that would otherwise be intractable because of their structural complexity, like the integrated circuits of our computers [26-39, 59-61], or the mitochondria of our cells and tissues [79-81].

The Maxwell current law eq. (3) derived from the Maxwell-Ampere equation eq. (1) is valid for any material with any type of polarization. It is universal, without adjustable parameters. The Maxwell current law depends on the properties of space-time and is valid in a vacuum and inside atoms. It—like the Maxwell-Ampere equation eq. (1) from which it is derived—does not depend on properties of matter.

Engineering systems Many of the most important applications of electrodynamics involve total current in complex systems and involve a large range of scales. The electric power systems that deliver energy to our homes and offices; the circuits of our telephones that deliver information everywhere; and the circuits of our computers [26-39, 59-61] all depend on the properties of total current. They are designed so that the details of the conduction currents are not very important. The properties of the total true current are very important. As Landauer [82] p. 112 put it, "It is, after all, the sum of electron current and displacement current which has no divergence. One of those two components can take over from the other."

High-speed transistors often have complex structures dominated by different current carriers at different times [61, 83-85]. The laws of total current greatly simplify their analysis. The total current in each part of series model of a transistor is equal at all times [82, 86, 87]. Integrated circuits of billions of transistors can be designed and understood without understanding all the conduction currents or current carriers [26-39, 59-61]. Maxwell's true current allows such analysis [88].

Ionic solutions play as wide a role in biology and chemistry as solid semiconductors play in engineering. The composition and concentration of ionic solutions have large effects on most biological and chemical systems. In biology, very large concentrations of ions (> 10 molar) are found near nucleic acids and in crucial locations in enzymes and ion channels. But trace concentrations (10^{-8} M) Ca^{2+} are often controllers of natural function. They, for example, control the contraction of skeletal and cardiac muscle.

Ionic solutions have important properties over a range of scales from atomic to macroscopic. The full range of time scales is needed to understand ionic solutions in biology:

1) The fastest time scales involve universal displacement currents (eq.(5)).
2) Fast time scales involve material displacement currents of the water, scaled by the Bjerrum constant.
3) Somewhat slower time scales involve rearrangements of the ionic atmosphere, scaled by the Debye length.
4) Time scales are associated with the movement of each type of ion. Each of the actual conduction currents of the ions themselves are scaled by its own Ohm's law, and its ionic resistivities.
5) Slower time scales come from the diffusion of mass of each ion and the solvent, scaled by Fick's law and its ionic diffusion constants.
6) The flow of the bulk of the solution ('osmosis' in the classical literature) is scaled by water permeability and water's diffusion constant and permeability.

Biological systems involve many scales. Understanding the biological role of ionic solutions requires using a large set of field equations—partial differential equations—with many parameters, more or less one set per scale. Each of the partial differential equations involves interaction coefficients. Such systems are difficult to specify, describe by models, solve, or compute.

Separate equations have been used for each scale but an integrated approach is obviously preferable and probably necessary. An integrated approach requires equations that are true on

all scales. Laws of total current that are universal are a substantial help when analyzing such systems because they dramatically reduce the number of equations and parameters. Universal current laws are multiscale and provide a link between atomic scale calculations and macroscopic behavior. They provide a firm foundation for building the complex theories needed to describe the complex systems of both engineering and life.

True Current in Life. Multiscale analysis is particularly important in living systems. The full range of scales of ionic solutions are involved in almost all living processes.

It is not possible to understand biology using only one set of scales. On the atomic scale, genes control biological function even though genes only involve small numbers of atoms, as the revolution of molecular biology has shown us. Genes control almost all biological function. The function of organs depends on atomic scales even though organs function on a macroscale. The kidney is a striking example. Mutations involving a handful of atoms in transporters in kidney tubules change water flow throughout the organ, and the animal as well, for that matter.

Some biological systems use total true current for their main biological output. The main biological output of many nerve cells is their electrical signal, the all-or-none binary signal called the action potential. The action potential propagates to carry signals in the nervous system and to coordinate contraction in muscle, including the heart. It is the action potential that allows the heart to function as a pump.

In nerve and muscle fibers and cells it is total current that flows down the length of the fiber and propagates the action potential signal. The components of conduction current vary down the length of the nerve fiber and the nature of current carrier does not matter for the propagation of the signal from node to node in myelinated fibers. What matters is just the total current.

Energy of Life. Total true currents are also involved in the generation of the chemical energy of life, ATP. ATP is the currency of chemical energy that powers life. Despite the fantastic diversity of biology, from virus to whale, ATP is the only source of chemical energy, with very few exceptions. More information can be found in many textbooks [79-81].

ATP is generated by the flow of total true current in organelles found inside cells called mitochondria and chloroplasts [79-81]. The membranes of these organelles contain molecular machines ('complexes', e.g., electron transport complexes I-IV in mitochondria). Each complex has different outputs, including conduction currents of electrons, ions, displacement current of the membrane capacitance, and conduction currents of protons—the biological name for positively charged water. The total current produced by these complexes follows the Maxwell current law. Mitochondria and chloroplasts have no length and are 'the opposite' of nerve fibers. Total current can flow down a long nerve fiber. Total current cannot flow down a mitochondria because mitochondria are short organelles. The total current that flows into them must also leave. Mitochondria have no long extension where current might flow.

Total current can NEVER accumulate, according to the Maxwell current law. Thus, the components of total currents crossing the membrane of mitochondria are coupled. When one component increases, the other must decrease. The components of current across mitochondrial membranes are linked by electrodynamics because of the Maxwell current law. That linkage is a consequence of universal physics, not a consequence of chemical or conformational or allosteric interactions.

The total current of the complexes in the membranes of the mitochondria is the total output of the metabolism of animal cells. Because the total current that flows into the mitochondrion also flows out, the total current is also the total current that flows out across the enzyme ATP synthase. The enzyme ATPsynthase produces ATP from the total current [89]. The total current of the complexes is linked to the total current of the ATPsynthase because they are both in the same membrane of a very short organelle, the mitochondrion. The linkage is physical, not conformational. Not allosteric. Not chemical. Maxwell's true current—not individual fluxes of protons, electrons, or ions—provides the energy that creates ATP, the chemical energy of life.

The total true current has simple behavior: It is the sum of its components, no matter how complex they are. Analysis of the sum is much easier than analyzing components. The components of conduction current are each very complex in mitochondria, for example. But the sum of the components is not complex. It follows a simple addition law, the Maxwell current law. A

simple addition law does not apply to the components individually. The addition law applies only to their sum, the total true current.

Analysis of ATP production in mitochondria is very difficult if each component must be understood in detail. Analysis of ATP production is greatly simplified if our main concern is the total true current that fuels ATPsynthase.

Notations

Maxwell often described the electric field with his displacement field $\mathbf{D} = \varepsilon_r \varepsilon_0 \mathbf{E}$, rather than with the $\mathbf{E}$ field itself shown in eq.(1). Modern textbooks have abandoned, with prejudice, the $\mathbf{D}$ field: p. 2-10 of [51]; Ch. 4 of [4]; p. 500-507 of [44]; p. 44, 160, of [52]; [55]. As Purcell and Morin (p. 500-507 of [44]) put it "the introduction of **D** is an artifice that is not, on the whole, very helpful. We have mentioned **D** because it is hallowed by tradition, beginning with Maxwell, and the student is sure to encounter it in other books, many of which treat it with more respect than it deserves."

The displacement field deserves no respect, in my view, because the displacement field mixes two quite different kinds of physics

$$\mathbf{D} = \overbrace{\varepsilon_0 \mathbf{E}}^{Universal} + \overbrace{(\varepsilon_r - 1)\varepsilon_0 \mathbf{E}}^{Material} = \varepsilon_r \varepsilon_0 \mathbf{E} \tag{5}$$

The universal term $\varepsilon_0 \mathbf{E}$ describes the properties of space—in a vacuum or inside an atom, or anywhere. The factor ε_0 is exact as far as has been measured and is the same in empty space (devoid of mass or charge) and in matter, whether inside a proton, subatomic particle, or a star.

The other term $(\varepsilon_r - 1)\varepsilon_0 \mathbf{E}$ is as different as one can imagine. It is a material term that varies enormously because it has to describe all the properties of charged matter. The material term is a constitutive equation that depends on the properties of matter in a remarkably complex way, as discussed in [14] and documented in [12, 13, 15-24]. The spectroscopic spectra and properties [15], p. 241-275 Section "L.2.4.A-E of matter best document the diversity of polarization [12, 16-23, 90-93]. The millions of compounds of organic chemistry are identified by their distinct spectroscopic properties, i.e., spectra. Thousands of dielectric parameters $\varepsilon_r(\omega)$ are needed to describe the polarization of charge at different frequencies ω in a single spectrum, and there are millions of spectra to deal with. A single ε_r is seriously misleading. It provides a short-sighted view of the realities of polarization.

If the Maxwell equations are written with the displacement $\mathbf{D}$ as a dependent variable, the general nature of the Maxwell equations is obscured by the particular nature of $\mathbf{P}$ different for each type of material, including millions of organic compounds. That is probably why Feynman, Griffiths, Purcell, and Morin, Ulaby, and Zangwill abandon the **D** field notation. That is why the **D** field is not used in the body of this paper.

Acknowledgement

It is a joy to thank Ardyth Eisenberg for her contributions to this paper and to so much else in my life.

References

1. Siegel, D. M. 2003. Innovation in Maxwell's electromagnetic theory: Molecular vortices, displacement current, and light. Cambridge University Press.
2. Bromberg, J. 1967. Maxwell's displacement current and his theory of light. Archive for History of Exact Sciences 4(3):218-234.
3. Cohen, R. S., and J. J. Stachel. 1979. The Velocity of Light and the Evolution of Electrodynamics [1956a]. Selected Papers of Léon Rosenfeld. Springer, pp. 134-177.
4. Griffiths, D. J. 2024. Introduction to Electrodynamics, Fifth Edition,. Cambridge University Press.
5. Jackson, J. D. 1999. Classical Electrodynamics, Third Edition. Wiley, New York.
6. Cohen, E. R., T. Cvitas, J. Frey, B. Holmstrom, K. Kuchitsu, R. Marquardt, I. Mills, F. Pavese, M. Quack, J. Stohner, H. L. Strauss, M. Takami, and A. J. Thor. 2007. Quantities, Units and Symbols in Physical Chemistry. Royal Society of Chemistry Publishing, Cambridge, UK.
7. Maxwell, J. C. 1865. A Treatise on Electricity and Magnetism (reprinted 1954). Dover Publications, New York.
8. Berry, S. R., S. A. Rice, and J. Ross. 2000. Physical Chemistry. Oxford, New York.
9. Laidler, K. J., J. H. Meiser, and B. C. Sanctuary. 2003. Physical Chemistry. BrooksCole, Belmont CA.
10. Atkins, P., and J. de Paula. 2010. Physical Chemistry.
11. Maxwell, J. C. 1865. A Dynamical Theory of the Electromagnetic Field. Philosophical Transactions of the Royal Society of London 155:459-512.
12. Barsoukov, E., and J. R. Macdonald. 2018. Impedance spectroscopy: theory, experiment, and applications. John Wiley & Sons.
13. Robinson, F. N. H. 1973. Macroscopic electromagnetism. Pergamon.
14. Eisenberg, R. S. 2019. Dielectric Dilemma. preprint available at https://arxiv.org/abs/1901.10805.
15. Parsegian, V. A. 2006. Van der Waals Forces: A Handbook for Biologists, Chemists, Engineers, and Physicists. Cambridge University Press, New York.
16. Banwell, C. N., and E. M. McCash. 1994. Fundamentals of molecular spectroscopy. McGraw-Hill New York.
17. Barthel, J., R. Buchner, and M. Münsterer. 1995. Electrolyte Data Collection Vol. 12, Part 2: Dielectric Properties of Water and Aqueous Electrolyte Solutions. DECHEMA, Frankfurt am Main.

18. Ciucci, F. 2018. Modeling Electrochemical Impedance Spectroscopy. Current Opinion in Electrochemistry.

19. Demchenko, A. P. 2013. Ultraviolet spectroscopy of proteins. Springer Science & Business Media.

20. Jaffé, H. H., and M. Orchin. 1962. Theory and applications of ultraviolet spectroscopy.624.

21. Kremer, F., and A. Schönhals. 2003. Broadband Dielectric Spectroscopy. Springer.

22. Rao, K. N. 2012. Molecular spectroscopy: modern research. Elsevier.

23. Sindhu, P. 2006. Fundamentals of Molecular Spectroscopy. New Age International.

24. Stuart, B. 2005. Infrared spectroscopy. Wiley Online Library.

25. Wang, Y., C. Liu, and B. Eisenberg. 2021. On variational principles for polarization in electromechanical systems. arXiv preprint arXiv:2108.11512.

26. Jerome, J. W. 1995. Analysis of Charge Transport. Mathematical Theory and Approximation of Semiconductor Models. Springer-Verlag, New York.

27. Selberherr, S. 1984. Analysis and Simulation of Semiconductor Devices. Springer-Verlag, New York.

28. Vasileska, D., S. M. Goodnick, and G. Klimeck. 2010. Computational Electronics: Semiclassical and Quantum Device Modeling and Simulation. CRC Press, New York.

29. Laux, S. E., and K. Hess. 1999. Revisiting the analytic theory of pn junction impedance: Improvements guided by computer simulation leading to a new equivalent circuit. IEEE Transactions on Electron Devices 46(2):396-412.

30. Horowitz, P., and W. Hill. 2015. The Art of Electronics. Cambridge University Press.

31. Horowitz, P., and W. Hill. 2020. The Art of Electronics: The x Chapters. Cambridge University Press.

32. Ayers, J. E. 2018. Digital Integrated Circuits: Analysis and Design, Second Edition. CRC Press.

33. Camenzind, H. 2005. Designing analog chips. Virtualbookworm Publishing.

34. Gielen, G., and W. M. Sansen. 2012. Symbolic analysis for automated design of analog integrated circuits. Springer Science & Business Media.

35. Gray, P. R., P. J. Hurst, S. H. Lewis, and R. G. Meyer. 2009. Analysis and Design of Analog Integrated Circuits. Wiley.

36. Hall, S. H., and H. L. Heck. 2011. Advanced signal integrity for high-speed digital designs. John Wiley & Sons.

37. Lienig, J., and J. Scheible. 2020. Fundamentals of layout design for electronic circuits. Springer Nature.

38. Scherz, P., and S. Monk. 2006. Practical electronics for inventors. McGraw-Hill, Inc.

39. Johnson, H. W., and M. Graham. 2003. High-speed signal propagation: advanced black magic. Prentice Hall Professional.

40. Eisenberg, R., and D. Chen. 1993. Poisson-Nernst-Planck *(PNP)* theory of an open ionic channel. Biophys J 64:A22.

41. Horng, T. L., T. C. Lin, C. Liu, and B. Eisenberg. 2012. PNP equations with steric effects: a model of ion flow through channels. J Phys Chem B 116(37):11422-11441.

42. Eisenberg, R. 2019. PNP what is in a name july 25-1 2019. pdf 10.31224/osf.io/2739d. engrXiv. August 3.

43. Xie, D., and Z. Chao. 2020. A finite element iterative solver for a PNP ion channel model with Neumann boundary condition and membrane surface charge. Journal of Computational Physics:109915.

44. Purcell, E. M., and D. J. Morin. 2013. Electricity and magnetism. Cambridge University Press.

45. Eisenberg, R. S. 2021. Maxwell equations without a polarization field, Using a paradigm from biophysics. Entropy 23(2):172.

46. Kreyszig, E. 2020. Advamced Emgineering Mathematics. WILEY, [S.l.].

47. Arfken, G. B., H. J. Weber, and F. E. Harris. 2013. Mathematical Methods for Physicists: A Comprehensive Guide. Elsevier Science.

48. Abraham, R., and J. E. Marsden. 1985. Foundations of Mechanics. Addison Wesley, New York.

49. Chorin, A. J., O. H. Hald, and R. Kupferman. 2000. Optimal prediction and the Mori-Zwanzig representation of irreversible processes. Proceedings of the National Academy of Sciences 97(7):2968-2973.

50. Sen, S., Y. Wang, T. Breitzman, and K. Dayal. 2026. Two-Scale Analysis of the Electrostatics of Dielectric Crystals: Emergence of Polarization Density and Boundary Charges. Multiscale Modeling & Simulation 24(1):162-186.

51. Feynman, R. P., R. B. Leighton, and M. Sands. 1963. The Feynman: Lectures on Physics, Vol. 2. Mainly Electromagnetism and Matter. Addison-Wesley Publishing Co., also at http://www.feynmanlectures.caltech.edu/II_toc.html, New York.

52. Zangwill, A. 2013. Modern Electrodynamics. Cambridge University Press, New York.

53. Panofsky, W. K. H., and M. Phillips. 2012. Classical Electricity and Magnetism: Second Edition. Dover Publications.

54. Jeans, J. H. 1908. The mathematical theory of electricity and magnetism. Cambridge University Press.

55. Ulaby, F. T., and U. Ravaioli. 2022. Fundamentals of Applied Electromagnetics, Global Edition. Pearson Education.

56. Kirchhoff, G. 1857. LIV. On the motion of electricity in wires. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 13(88):393-412.

57. Oriols, X., R. Eisenberg, and D. K. Ferry. 2025. Kirchhoffs Forgotten Contributions to Electromagnetism: Continuity Equation versus Displacement Currents. arXiv preprint arXiv:2512.05997.

58. Ferry, D. 2019. The Copenhagen Conspiracy. Jenny Stanford Publishing.

59. Ferry, D. K. 2020. Quantum Mechanics: An Introduction for Device Physicists and Electrical Engineers. Taylor & Francis Group.

60. Oriols, X., and D. K. Ferry. 2021. Why engineers are right to avoid the quantum reality offered by the orthodox theory?[point of view]. Proceedings of the IEEE 109(6):955-961.

61. Ferry, D. K., X. Oriols, and J. Weinbub. 2023. Quantum Transport in Semiconductor Devices. In Simulation using particles. IOP Publishing.

62. Macleod, A. J., J. P. Edwards, T. Heinzl, B. King, and S. V. Bulanov. 2023. Strong-field vacuum polarisation with high energy lasers. New Journal of Physics 25(9):093002.

63. Scharnhorst, K. 2017. Photon-photon scattering and related phenomena. Experimental and theoretical approaches: The early period. arXiv preprint arXiv:1711.05194.

64. Whittaker, E. 1951. A History of the Theories of Aether & Electricity. Harper, New York.

65. Einstein, A., H. A. Lorentz, and H. Weyl. 2013. The principle of relativity. Courier Corporation.

66. Janssen, M., and C. Lehner. 2014. The Cambridge Companion to Einstein. Cambridge University Press.

67. Brown, H. R. 2005. Physical relativity: Space-time structure from a dynamical perspective. Oxford University Press.

68. Resnick, R. 1991. Introduction to special relativity. John Wiley & Sons.

69. Abraham, M., and R. Becker. 1932. The Classical Theory of Electricity and Magnetism. Blackie and subsequent Dover reprints, Glasgow, UK.

70. Balanis, C. A. 2012. Advanced engineering electromagnetics. John Wiley & Sons.

71. Lorrain, P., D. R. Corson, and F. Lorrain. 1988. Electromagnetic Fields and Waves: Including Electric Circuits. Freeman.

72. Lu, B., X. Cheng, J. Huang, and J. A. McCammon. 2006. Order N algorithm for computation of electrostatic interactions in biomolecular systems. Proceedings of the National Academy of Sciences of the United States of America 103(51):19314-19319.

73. Eisenberg, R. S. 2025. Kirchhoff's Current Law: A Derivation from Maxwell's Equations. Computation 13(8):200.

74. Eisenberg, R. S. 2020. Electrodynamics Correlates Knock-on and Knock-off: Current is Spatially Uniform in Ion Channels. Preprint on arXiv at https://arxiv.org/abs/2002.09012.

75. Eisenberg, R. S. 2025. Truly Incompressible: Maxwell's Total Current.

76. Eisenberg, B. 2014. Shouldn't we make biochemistry an exact science? ASBMB Today 13(9, October):36-38, Available on arXiv as https://arxiv.org/abs/1409.0243.

77. Popper, K. 2005. The logic of scientific discovery. Routledge.

78. Kuhn, T. S. 1997. The structure of scientific revolutions. University of Chicago press Chicago.

79. Darnell, J., H. Lodish, and D. Baltimore. 1990. Molecular Cell Biology. Scientific American Books, New York.

80. Alberts, B., D. Bray, J. Lewis, M. Raff, K. Roberts, and J. D. Watson. 1994. Molecular Biology of the Cell. Garland, New York.

81. Berg, J. M., J. L. Tymoczko, and L. Stryer. 2010. Biochemistry. W. H. Freeman, New York.

82. Landauer, R. 1992. Conductance from transmission: common sense points. Physica Scripta 1992(T42):110.

83. Villani, M., X. Oriols, S. Clochiatti, N. Weimann, and W. Prost. 2020. The accurate predictions of THz quantum currents requires a new displacement current coefficient instead of the traditional transmission one. In 2020 Third International Workshop on Mobile Terahertz Systems (IWMTS). IEEE. 1-5.

84. Destefani, C. F., M. Villani, X. Cartoixà, M. Feiginov, and X. Oriols. 2022. Resonant tunneling diodes in semiconductor microcavities: Modeling polaritonic features in the terahertz displacement current. Physical Review B 106(20):205306.

85. Villani, M., S. Clochiatti, W. Prost, N. Weimann, and X. Oriols. 2021. There is plenty of room for THz tunneling electron devices beyond the transit time limit. IEEE Electron Device Letters 42(2):224-227.

86. Landauer, R. 1996. Mesoscopic noise: Common sense view. Physica B: Condensed Matter 227(1-4):156-160.

87. Imry, Y., and R. Landauer. 1999. Conductance viewed as transmission. Reviews of Modern Physics 71(2):S306.

88. Eisenberg, R. S. 2023. Maxwell's Current in Mitochondria and Nerve. arXiv:http://arxiv.org/abs/2309.05667.

89. Eisenberg, R. S. 2025. Current Flow in Nerves and Mitochondria: An Electro-Osmotic Approach. Biomolecules 15(8):1063.

90. Mellor, B. L., E. C. Cortes, D. D. Busath, and B. A. Mazzeo. 2011. Method for estimating the internal permittivity of proteins using dielectric spectroscopy. J Phys Chem B 115(10):2205-2213.

91. Buchner, R., and J. Barthel. 2001. Dielectric Relaxation in Solutions. Annual Reports on the Progress of Chemistry, Section C: Physical Chemistry 97:349-382.

92. Barthel, J., H. Krienke, and W. Kunz. 1998. Physical Chemistry of Electrolyte Solutions: Modern Aspects. Springer, New York.

93. Angulo-Sherman, A., and H. Mercado-Uribe. 2011. Dielectric spectroscopy of water at low frequencies: The existence of an isopermitive point. Chemical Physics Letters 503(4–6):327-330.