Previous investigators who have concluded that the transit time of positive ions in their cells was too short to account for the lags they observed, have generally neglected to calculate the potential distribution in the cell. Without this knowledge such a conclusion is questionable.

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Currents to Conductors Induced by a Moving Point Charge

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General expressions are derived for the currents which flow in the external circuit connecting a system of conductors when a point charge is moving among the conductors. The results are applied to obtain explicit expressions for several cases of practical interest.

In the earlier days of vacuum tube technique when the radiofrequencies in use were relatively low compared to those attained at present, it was acceptable to regard the transit of an electron across a vacuum tube as an instantaneous burst of current. At present, however, the time of transit of the electron is of comparable duration with the periods of alternating circuits and it is consequently of interest to know the instantaneous value of the current induced by the moving charge over its entire time of transit.

Before discussing what effect the moving charge has, we must introduce certain conventions as to what part of the total field is to be attributed to the charge and what part to other causes. It proves most convenient to consider that all of the conductors are grounded and to examine the currents to them through the external circuit due to the motion of the charge. If the voltages on the conductors are varying, however, charges will be induced and currents will flow as dictated by the coefficients of capacity. In keeping with the superposition principle,[1] the net current is found by adding the currents induced by the moving charge (or each moving charge if there are several) and the currents due to changing voltages.

[1] Jeans, *Mathematical Theory of Electricity and Magnetism*, fourth edition, p. 90.

We are thus led to consider the charges and currents induced on a system of grounded stationary conductors by the motion of a point charge. If we have a system of grounded conductors, perhaps as illustrated in Fig. 1, num-

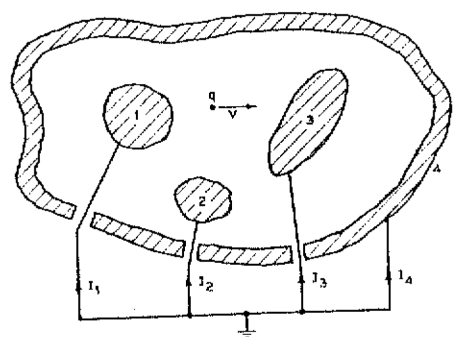

FIG. 1. Schematic representation of conductors and currents.

bered 1, 2, $\cdots$ n, say, then the charge q_1 induced on conductor 1 due to a unit point charge in the space is calculated as follows: Let conductor 1 be at unit potential and the others be grounded and let the space between the conductors be free of charge. The electrostatic potential produced by this situation has the value $V_1(r)$ at the arbitrary point r of space. Then, in terms of this potential distribution, the charge, q_1, induced on 1 by a unit charge at r is

$$q_1 = -V_1(r). \qquad (1)$$

Proof: Let us imagine that a vanishingly small conductor is placed at r; let it be number 0 and the other conductors be numbers 1 to n.

Situation 1. Let conductor 0 be uncharged, conductor 1 be at unit potential, $2 \cdots n$ be at zero potential. The charges Q_i' and potentials V_i' accordingly satisfy the equations: $V_0' = V_1(r)$, $V_1' = 1$, $V_2' = \cdots = V_n' = 0$; $Q_0' = 0$. *Situation 2.* Let conductor 0 possess unit charge. Let all other conductors be grounded and denote the charge on 1 by q_1. Then $Q_0'' = 1$, $Q_1'' = q_1$, $V_1 = \cdots = V_n = 0$. According to Green's reciprocation theorem:[2]

$$0 = \Sigma_i (V_i' Q_i'' - V_i'' Q_i') = V_1(r) + q_1$$

and this is Eq. (1).

If the point charge moves with a vector velocity $v = dr/dt$, we find for I_1, the current which flows to conductor 1 through the external circuit,

$$I_1 = dq_1/dt = -\nabla V_1(r) \cdot dr/dt = E_1(r) \cdot v, \quad (2)$$

where $E_1(r)$ is the electric field at point r due to unit potential on 1 with $2 \cdots n$ grounded. By the same argument we find that the current to the ith conductor is

$$I_i = E_i(r) \cdot v. \quad (3)$$

(As was pointed out above, the current just calculated, although it is expressed in terms of fields produced by potentials on the electrodes, is that induced by the motion of the charge and does not include currents produced by changing potentials upon the conductors.) Since we have used a theorem of electrostatics in our theory, the results will not be valid if retardation effects are important within the volume throughout which the charges move; they will be valid, however, if these effects are small, even for large transit angles of the moving charges.

[2] Reference 1, p. 92.

Applications

Infinite plane parallel plates

If conductors 1 and 2 are perpendicular to the X axis and have intercepts $x=0$ and $x=d$, respectively, then $E_1 = 1/d$ and $E_2 = -1/d$; and for a velocity $v = dx/dt$, we get the data of Table I.

TABLE I. *Currents induced by unit point charge moving with velocity v.*

	I_1	I_2	I_3
Parallel Planes	v/d	$-v/d$	—
Coaxial Cylinders	$v/r \ln b/a$	$-v/r \ln (b/a)$	—
Concentric Spheres	$\frac{vab}{(b-a)r^2}$	$-\frac{vab}{(b-a)r^2}$	—
Cylindrical Triode $a<r<b$	$\frac{v}{r \ln b/a}$	$\frac{-\mu v}{(1+\mu) r \ln (b/a)}$	$\frac{-v}{(1+\mu) r \ln (b/a)}$
$b<r<c$	0	$\frac{\mu v}{(1+\mu) r \ln c/b}$	$\frac{-\mu v}{(1+\mu) r \ln (c/b)}$

Coaxial cylinders and concentric spheres

For these we let 1, the inner conductor, have radius a and 2 have radius b and let $v = dr/dt$.

Cylindrical triode

Let the three electrodes in order of increasing radii be 1, cathode, radius a; 2 grid b; 3 anode c. Denoting the amplification factor by μ and assuming that $\mu \gg 1$, the average potentials at the grid due to unit potential on 1, 2, and 3, respectively, are $V_1(b) = 0$, $V_2(b) = \mu/(1+\mu)$, $V_3(b) = 1/(1+\mu)$.[3] Letting $v = dr/dt$, we get the remaining data of Table I.

[3] For a more accurate treatment suitable for small amplification factors see Dow, *Fundamentals of Engineering Electronics*, p. 49.